

Purcell effect in a magnetic cavity

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Abstract: We study dipole emission inside a homogeneous, magnetic sphere. For fixed refractive index, the largest emission rate for an *electric* source is observed, in general, when the *magnetic* permeability is maximized. The corresponding result for a magnetic source follows by symmetry.

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It is well-established that the radiation of sources depends on the physical environment in the vicinity of the source [1]. In particular, a source placed in an electromagnetic cavity can have its emission enhanced or suppressed, depending on the cavity properties. Until recently it was thought possible to tune only the electric properties of the cavity (via the permittivity ϵ), however metamaterials offer the possibility of controlling both the electric and magnetic responses of a medium, leading to markedly different radiation dynamics [2]. We study here the emission of sources placed in metamaterial cavities, in which the magnetic properties may be controlled by changing the permeability μ . We investigate the circumstances in which emission may be enhanced or suppressed, and outline how the inclusion of magnetic effects leads to differences in the physics of radiating sources.

Consider a classical oscillating electric dipole, with normalized dipole moment \mathbf{p} , located at position \mathbf{r}_0 in vacuum. We denote by Γ_∞ the rate at which the dipole radiates energy. If we now consider the same dipole placed in a given environment, the emission rate changes to Γ_{env} given by

$$\frac{\Gamma_{\text{env}}}{\Gamma_\infty} = 1 + \frac{3}{2k_0^3} \mathbf{p} \cdot \text{Im}[\bar{\mathcal{G}}_{\text{env}}(\mathbf{r}_0, \mathbf{r}_0; \omega)] \cdot \mathbf{p}, \quad (1)$$

where $\bar{\mathcal{G}}_{\text{env}}$ is the field-susceptibility tensor (FST) [3] associated with the environment, and k_0 is the vacuum wavenumber.

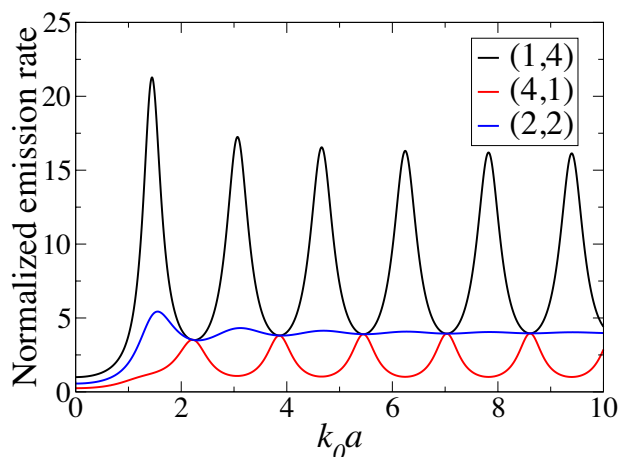


Fig. 1. Normalized emission rate for an electric dipole at the center of a spherical cavity (radius a , refractive index $n = \sqrt{\epsilon\mu} = 2$) in vacuum versus $k_0 a$. The inset gives the value of (ϵ, μ) .

Classically, the above expression gives the normalized power losses for an oscillating electric dipole. In the quantum-mechanical picture, the same expression corresponds to the spontaneous emission rate for an electric dipole transition

in the weak-coupling regime. Note that Eq. (1) applies to lossy, anisotropic and dispersive media, provided the material response is linear in the fields. A similar expression can be derived for a magnetic dipole source, or deduced from the electric case using symmetry arguments. Therefore, provided one knows the FST associated with a given structure, one can, in principle, study the dynamics of a source placed inside the structure.

One of the simplest cases of a magnetic cavity is a lossless homogeneous, isotropic, spherical magnetodielectric cavity in vacuum, with a radius a and refractive index $n = \sqrt{\epsilon\mu}$, where ϵ and μ are the positive (scalar) permittivity and permeability, respectively. For this system, the corresponding FST can be calculated analytically using Chew's approach [4]. In Fig. 1 we plot the emission rate, normalized to free-space, for an electric dipole source at the center of the cavity versus the normalized cavity size k_0a . The refractive index of the cavity is fixed at $n = 2$ and the value of the pair (ϵ, μ) is given in the inset. As expected the emission rate oscillates as we increase the size of the cavity introducing additional resonant modes. In the case $\epsilon = \mu = 2$ the oscillations are quickly damped because the impedance of the material inside the cavity matches the impedance of vacuum. However, the most interesting feature is that the emission rate for the electric source is in general maximized when μ is maximized, and therefore since n is fixed, when ϵ is minimized. Hence the emission rate of the electric source is in general most enhanced when the magnetic response of the cavity is maximized. For a magnetic source, the roles of ϵ and μ are exchanged. This is due to the asymmetric role played by ϵ and μ in the emission rate of an electric source—for instance, for an infinite medium with refractive index n the density of states scales as $n^3 = (\epsilon\mu)^{3/2}$, whereas the electric field scales as $1/\sqrt{\epsilon} = \sqrt{\mu}/n$.

The results are slightly different if the source is moved off-center, as seen in Fig. 2 for an electric dipole source located $0.6a$ from the center. We must now consider both radial and tangential orientations of the dipole moment, however, while the values of the emission rate are different, the two cases are qualitatively similar. For a small cavity ($k_0a \lesssim 5$) the results are similar to the centered-source case: the emission rate is maximized for an electric (magnetic) source when μ (ϵ) is maximized. However, as the cavity gets larger, sharp resonances appear corresponding to the coupling of the source to high-Q modes inside the sphere. At certain frequencies, the emission rate for $(\epsilon, \mu) = (4, 1)$ can now exceed the rate for $(\epsilon, \mu) = (1, 4)$. Nonetheless, the overall picture remains that for a given refractive index, the emission rate of an electric source is most enhanced when the magnetic response of the material is maximized. For a magnetic source the opposite holds.

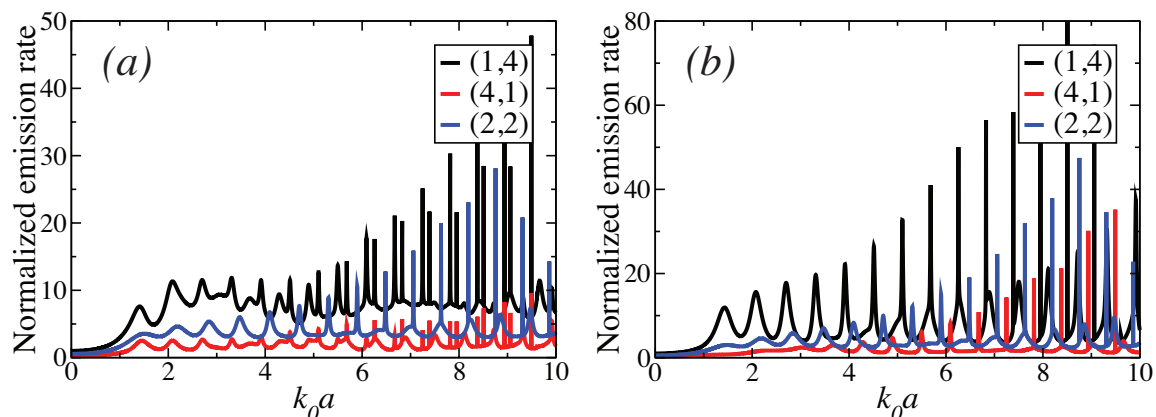


Fig. 2. As for Fig. 1 for a source offset $0.6a$ from the center of the cavity. (a) radial dipole, (b) tangential dipole.

Finally, using the discrete dipole approximation [2], we have found that discrete cavities alter this picture by introducing local-field corrections that depend on ϵ and μ in a radically different way. The dimensional dependence of the local density of states also means that different results are found for the corresponding two-dimensional problem. Both these facts will play a critical role when the influence of the microstructure of metamaterials on the emission rate is explicitly considered.

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