Current driven instability in collisional dusty plasmas

B. P. Pandey,[1,a)] S. V. Vladimirov,[2] and A. Samarian[2]
[1]Department of Physics, Macquarie University, Sydney, New South Wales 2109, Australia
[2]Complex Systems, School of Physics, The University of Sydney, New South Wales 2006, Australia

(Received 16 September 2009; accepted 26 October 2009; published online 18 November 2009)

The current driven electromagnetic instability in a collisional, magnetized, dusty medium is considered in the present work. It is shown that in the presence of the magnetic field aligned current, the low-frequency waves in the medium can become unstable if the ratio of the current to the ambient field is larger than the light speed times the wave number. The growth rate of the instability depends upon the ratio of the Alfvén to the dust cyclotron frequency as well as on the ratio of the current density J to the dust charge density Zen_d, where Z is the number of electronic charge on the grain, e is the electron charge, and n_d is the dust number density. The typical growth rate of this instability is on the order of Alfvén frequency which compares favorably with the electrostatic, cross-field current driven, Farley–Buneman instability and thus could play an important role in the Earth's ionosphere. © *2009 American Institute of Physics*. [doi:10.1063/1.3264834]

I. INTRODUCTION

The role of the charged grains in the space environment has been actively investigated for past several decades. It is believed that the dynamics of the charged dust is important to the stars and planet formation.[1] Investigations of the dust dynamics in our solar system, viz., spokes in the Saturn's ring, the Jovian ring formation, and formation of the noctilucent clouds in the Earth's mesosphere, are relatively recent.[2–6] One of the principal difficulties in studying the dynamics of the dusty plasma is associated with the complexity of the problem related to the wide distributions in the size, charge, and mass of the dust particles. In the space and astrophysical environments one generally finds graphite, silicate, and the metallic grains whose size can vary between a few angstroms to a few centimeters or more and charge can vary between 1 and 2 to hundreds of electrons.[1,3] The mass of the grain may vary as well between a few proton mass to a few billion proton mass. This wide distribution in the size, mass, and charge of the grain has profound implication on the dynamics of the dusty plasma. For example, for a singly charged grain, having $\sim 10^{-6}$ g mass and ~ 0.01 cm radius, the electrostatic and self-gravitational forces acting on the grain can balance each other.[7,8] This may affect the dynamics of self-gravitating dusty clouds. The balance between the gravity and the electrostatic force on the grain surface causes a thin cloud of dust over the semiconductor chips, posing considerable difficulty in the chip manufacturing.[4,9] Since charge on the grain can be manipulated by manipulating the ambient plasma conditions, they can be utilized as a probe to investigate the plasma sheath properties.[10] Therefore, the grain mass and charge variation modify the collective plasma behavior.[11,12]

The charge on the grain is often (but not always) determined by the collision with the plasma (electron and ion) particles.[13] Therefore, a self-consistent set of magnetized dusty plasma equations requires inclusion of the collision terms in the dynamics. Clearly, dusty plasma dynamics, depending upon the physical properties of the system, can be studied in various limits. For example, when the dynamical response time is much larger than the collisional time scale valid for the low frequency fluctuations, the multicomponent electron, ion, and dust system of equations can be reduced to a single fluid description[14] and the set of equations becomes similar to the Hall magnetohydrodynamics (MHD), except now that the origin of the Hall term is due to the relative drift between the plasma and the dust particles.[15] In the opposite limit, when the dynamic response time is much shorter than the collisional time scale, the inert dust cannot participate in the high frequency plasma modes, except causing considerable damping of the waves. In the present work, we shall assume submicron sized grains ($a \sim 10^{-5}$ cm) with $m_d \sim 10^{-15}$ g and utilize the low frequency, single fluid description of the dusty plasma similar to Ref. 14. This is motivated by the fact that the micron sized grains are believed to be responsible for the low frequency fluctuations in the planetary and interstellar medium.[16–20]

In the partially ionized dust free plasma consisting of the electrons, ions, and the neutrals, the Hall field can arise if the gyration of the ions is inhibited by the neutrals, i.e., $\omega_{ci} \ll \nu_{ni}$, where $\omega_{cj} = eB/m_j c$ and ν_{nj} is the cyclotron and collision frequencies of the jth particle and e, B, m_j and c are electric charge, magnetic field, mass, and speed of light, and the electron gyrates uninhibited across the field $(\omega_{ce} \gg \nu_{ne})$.[21] In the dusty plasma, it is realistic to assume that the bulk velocities of the plasma particles are comparable over the dust dynamical time scale, and thus the relative drift between the plasma particles and dust will cause the Hall field.[14] The charge separation and the ensuing Hall field owe their existence to the large dust inertia. Therefore, the Hall field is a characteristic of the magnetized dusty plasma.[15]

The current driven instability has a reach history, par-

[a)]Also at Complex Systems, School of Physics, University of Sydney, New South Wales 2006, Australia. Electronic mail: bpandey@physics.mq.edu.au.

ticularly in the context of explaining the irregularities in the ionosphere.[22–25] It is believed that the strong Hall current called electrojets, which flows perpendicular to the magnetic equator at an altitude of ~ 100 km in the ionosphere, is unstable. The Hall current instability (also called Farley–Buneman instability) has been observed around the equatorial zone or within the polar cap.[26] The role of the dust in the Hall current instability has been investigated[27–29] following the observations of the dusty layer below the equatorial electrojet region.[3] Note that the Hall current instability is electrostatic in nature where the magnetic field fluctuations are completely ignored. Further, the ambient current in this instability flows across the magnetic field. Therefore, this electrostatic instability, which requires a cross field current, treats the magnetic field as passive dynamical quantity, notwithstanding the observed daily variation in the magnetic fields in the narrow belt of the geomagnetic equator attributed mainly to the presence of dynamo currents at 90–120 km. Therefore, in the present work we shall investigate the current driven electromagnetic instability with the ambient current aligned to the background magnetic field. Clearly, the physical nature of the problem considered here is quite different from the past work on the Farley–Buneman instability in the dusty plasma.[28,29]

We shall assume that the medium is weakly ionized (for example, noctilucent cloud where the plasma density is much smaller than the neutral density) and the inertia of the plasma component is mainly due to the presence of the charged grains, i.e., $\rho_e \ll \rho_i \ll \rho_d$ (where $\rho_j = m_j n_j$ is the mass density with m_j as the mass and n_j as the number density of the jth particle). Although neutral particles will as well affect the dynamics by causing the Hall field over the neutral-ion collision time scale, we are concerned here with the extremely low frequency fluctuations, and thus neglect the role of the neutrals. Thus, we shall investigate the dynamics of dusty plasma consisting of electrons, ions, and the charged dust only. Although the charge fluctuation is responsible for the novel collective behavior, it does not affect the low frequency normal mode.[16] Hence, we shall neglect the charge dynamics and assume constant dust charge. The basic set of equations is given in Sec. II. Assuming equality between the electron and ion bulk velocities, the multifluid description is reduced to a single fluid description. This single fluid description is similar to the Hall MHD equations of a two component, fully ionized plasma. Notwithstanding this similarity, the nature of the Hall term is quite different in the dusty plasmas.[14,15] In Sec. III, the basic formulation of the problem is given and the dispersion relation is derived. The numerical roots of the dispersion relation are presented and the dependence of the instability on various parameters is discussed. In Sec. IV, possible application of the result with a brief summary is presented.

II. BASIC MODEL

Our model of a multicomponent dusty plasma consists of the electrons, ions, and the charged grains. The continuity equation for the electrons, ions, and the dust is

$$\frac{\partial \rho_j}{\partial t} + \nabla \cdot (\rho_j \mathbf{v}_j) = 0. \tag{1}$$

Here ρ_j is the mass density and $\mathbf{v}_j$ is the velocity, and j stands for electrons, ions, and grains. The momentum equations are

$$0 = -\nabla P_e - e n_e (\mathbf{E} + \mathbf{v}_e \times \mathbf{B}) - \rho_e \nu_{ed} (\mathbf{v}_e - \mathbf{v}_d), \tag{2}$$

$$0 = -\nabla P_i + e n_i (\mathbf{E} + \mathbf{v}_i \times \mathbf{B}) - \rho_i \nu_{id} (\mathbf{v}_i - \mathbf{v}_d), \tag{3}$$

$$\rho_d \frac{d\mathbf{v}_d}{dt} = -\nabla P_d + Zen_d (\mathbf{E} + \mathbf{v}_d \times \mathbf{B}) + \sum_{j=e,i} \rho_j \nu_{jd} (\mathbf{v}_j - \mathbf{v}_d). \tag{4}$$

The electron and ion inertia has been neglected while writing Eqs. (2) and (3). This is motivated by the fact that the inertia in the dusty plasma is largely carried by the dust grains. The momentum equations (2)–(4) are closed by assuming an isothermal equation of state $P_j = C_s^2 \rho_j$, where $C_s = \sqrt{T_j/m_j}$ is the sound speed. Equations (2)–(4) on the right hand side have the pressure gradient term, Lorentz force term with $\mathbf{E}$ and $\mathbf{B}$ as the electric and magnetic fields, respectively, e is the electric charge, Z is the number of charge on the grain, n_j is the number density and the collisional momentum exchange term, where $\nu_{jd} = n_d \langle \sigma v \rangle_{jd}$ is the collision frequency of the dust with jth species.

We shall define the mass density of the bulk fluid as $\rho = \rho_e + \rho_i + \rho_d \approx \rho_d$. Then the bulk velocity $\mathbf{v} = (\rho_i \mathbf{v}_i + \rho_e \mathbf{v}_e + \rho_d \mathbf{v}_d)/\rho \approx \mathbf{v}_d$. The continuity equation [summing up Eq. (1)] for the bulk fluid becomes

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0. \tag{5}$$

The momentum equation can be derived by adding Eqs. (2)–(4),

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla P + \frac{\mathbf{J} \times \mathbf{B}}{c}. \tag{6}$$

Here $P = P_e + P_i + P_d$ is the total pressure.

Making use of plasma quasineutrality condition $n_e = n_i + Zn_d$ in the current density $\mathbf{J} = e(n_i \mathbf{v}_i - n_e \mathbf{v}_e + Zn_d \mathbf{v}_d)$, and assuming $\mathbf{v}_e \simeq \mathbf{v}_i$, the relative drift between the electrons and dust velocities can be written as

$$\mathbf{v}_e - \mathbf{v}_d \simeq -\mathbf{J}/eZn_d. \tag{7}$$

Defining $\beta_j = \omega_{cj}/\nu_{jd}$ as the plasma Hall parameter—a measure of magnetization of the electrons and ions, we see that when plasma cyclotron frequency ω_{cj} dominates the plasma-dust collision frequency ν_{jd}, the Hall term ($\sim \mathbf{E} \times \mathbf{B}$) will dominate in Eqs. (2) and (3). Thus the Hall field is generated over the plasma-cyclotron time scale. Assuming that the electrons are strongly magnetized, i.e., $\beta_e \gg 1$ (valid in the lower E region of the Earth), and taking curl of the electron momentum equation (2), the induction equation, after making use of the Maxwell's equation, can be written as

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left[(\mathbf{v}_d \times \mathbf{B}) - \left(\frac{\mathbf{J} \times \mathbf{B}}{Zen_d} \right) \right]. \quad (8)$$

While writing Eq. (8), the electric field due to pressure gradient term in Eq. (2) has been neglected. In a weakly ionized dusty medium, the contributions of the plasma pressure terms are generally small. We shall utilize Eqs. (5), (6), and (8) along with an isothermal equation of state to investigate the dusty plasma dynamics.

III. THE EQUILIBRIUM AND DISPERSION RELATION

We assume that the plasma is immersed in a uniform background magnetic field $\mathbf{B}=(0,0,B)$. Often, the plasma inhomogeneities coupled with the field aligned current are thought to be responsible for the wide range of waves and fluctuations in the Earth's ionosphere.[30] The field aligned current could have its origin in the relative plasma motion. Although the role of the dust in the field aligned current and, subsequently, in the magnetosphere-ionosphere coupling is not clear, in the Jovian and Saturn's magnetosphere-ionosphere coupling, dust will play an important role. In the laboratory plasmas, the presence of field aligned current may cause the lowering of the threshold for the lower hybrid waves.[31] Thus, we shall assume an equilibrium current $\mathbf{J}=(0,0,J)$ aligned to the ambient magnetic field.

The linearized equations are

$$\frac{\partial \delta\rho}{\partial t} + \nabla \cdot \rho \delta\mathbf{v} = 0, \quad (9)$$

$$\rho \frac{\partial \delta\mathbf{v}}{\partial t} + C_s^2 \nabla \delta\rho = \frac{1}{c}(\delta\mathbf{J} \times \mathbf{B} + \mathbf{J} \times \delta\mathbf{B}), \quad (10)$$

$$\frac{\partial \delta\mathbf{B}}{\partial t} = \nabla \times \left[\delta\mathbf{v} \times \mathbf{B} - \frac{1}{Zen_d}(\delta\mathbf{J} \times \mathbf{B} + \mathbf{J} \times \delta\mathbf{B}) \right]. \quad (11)$$

While writing Eq. (10), we have assumed $\delta P = C_s^2 \delta\rho$. We shall assume that the waves are propagating along the background magnetic field, i.e., along the z. Thus Fourier transforming the perturbed quantities as $\exp(-i\omega t + ikz)$, the linearized momentum equation can be reduced to the following form:

$$\begin{pmatrix} \omega & 0 \\ 0 & \omega \end{pmatrix} \delta\mathbf{v} = \frac{1}{\rho} \begin{pmatrix} \frac{-kB}{4\pi} & \frac{-iJ}{c} \\ \frac{-iJ}{c} & \frac{-kB}{4\pi} \end{pmatrix} \delta\mathbf{B}. \quad (12)$$

We note that the pressure term has dropped out on the right hand side in the above equation. This is not surprising since although the medium is not cold, the wave propagating along the magnetic field, which is transverse mode, gets decoupled from the longitudinal acoustic mode. This can also be seen by dotting the Fourier transformed Eq. (10) with $\mathbf{k}$ which will give $k \cdot \delta\mathbf{v}=0$, which implies $\delta v_z=0$. Therefore, the fluid is effectively incompressible.

The linearized induction equation (11) becomes

$$\begin{pmatrix} i\left(\omega + \frac{kJ}{Zen_d}\right) & \frac{ck^2B}{4\pi Zen_d} \\ \frac{-ck^2B}{4\pi Zen_d} & -i\left(\omega + \frac{kJ}{Zen_d}\right) \end{pmatrix} \delta\mathbf{B} = ikB \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \delta\mathbf{v}. \quad (13)$$

The following dispersion relation can be derived from Eqs. (12) and (13):

$$\omega^4 + 2ak\omega^3 + (a^2k^2 - 2\omega_A^2 - b^2k^4)\omega^2 + C_0 = 0, \quad (14)$$

where

$$C_0 = \omega_A^4 \left[1 - \left(\frac{4\pi J}{cB} \right)^2 \frac{1}{k^2} \right], \quad (15)$$

$a=J/Zen_d$, $b=cB/(4\pi Zen_d)$, and $\omega_A \equiv kv_A$ is the Alfvén frequency with $v_A=B/\sqrt{4\pi\rho_d}$ representing the Alfvén velocity in the dusty fluid. In the absence of current term, i.e., $a=0$, Eq. (14) reduces to

$$\omega^2 = \omega_A^2 \pm \left(\frac{\omega_A^2}{\omega_{cd}} \right) \omega, \quad (16)$$

which for the short wavelength $(\omega_{cd} \ll \omega_A)$ fluctuations describes the whistler waves in the high frequency $(\omega_A \ll \omega)$ limit,

$$\omega \cong \frac{\omega_A^2}{\omega_{cd}}, \quad (17)$$

and the electrostatic $(\nabla \times \delta\mathbf{E} \approx 0)$ ion-cyclotron wave in the low frequency $\omega \ll \omega_A$ limit,

$$\omega \cong \omega_{cd}. \quad (18)$$

In the long wavelength limit, i.e., $\omega_A \ll \omega_{cd}$, we recover usual Alfvén wave $\omega^2=\omega_A^2$.

In the presence of the equilibrium current, the necessary condition for the instability $C_0<0$ reads

$$J > \frac{cB}{4\pi}k, \quad (19)$$

implying that no matter how weak is the strength of the ambient field aligned current density, the above condition can always be satisfied for the long wavelength fluctuations, albeit with small growth rate. Therefore, we may infer that the low frequency fluctuations in such a medium will always be susceptible to the field aligned current driven instability. In the Earth's ionosphere, where the current is due to the relative drift between the plasma particles, assuming $|J| \approx en_e v_e$, we see that for the typical drift speed of 5 km/s and $n_e \sim 3\times 10^3$ cm^{-3}, the current density $J \sim 2$ A/km^2. Such current density compares favorably with the observed electrojet value at ~90 km.[32] Assuming the field strength $B=0.3$ G, we see that the long wavelength fluctuations $\gtrsim 10^4$ km are unstable to the instability. The local value of the current density J determines the cut-off wavelength below which the waves will remain stable.

We solve Eq. (14) numerically. The real and imaginary parts of the ω/ω_A against $4\pi J/cBk$ are shown in Fig. 1(a) for the fixed $J/(Zen_d)=0.1v_A$ and $\omega_A^2=0.01\omega_{cd}^2$. Choice of the

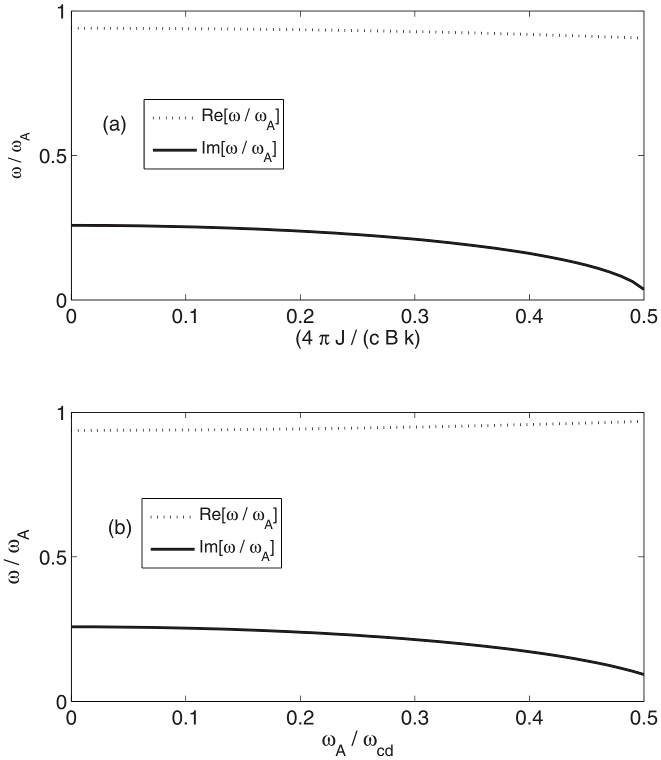

FIG. 1. The real and imaginary parts of the ω/ω_A against $4\pi J/cBk$ are shown in (a) for fixed $J/(Zen_d)=0.1v_A$ and $\omega_A^2=0.01\omega_{cd}^2$. In (b) real and imaginary parts of the ω/ω_A are plotted against ω_A/ω_{cd} for fixed $J/Zen_d=0.1v_A$ and $4\pi J/cBk=0.01$.

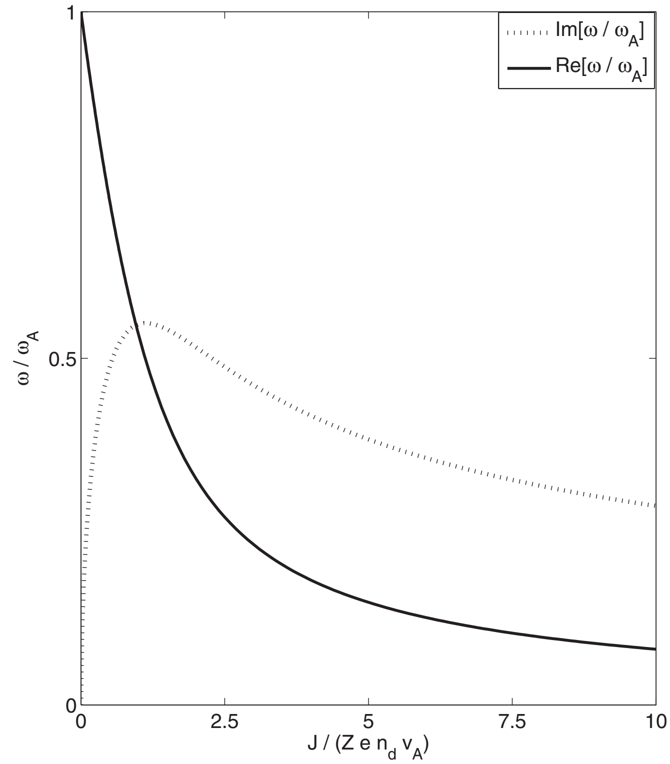

FIG. 2. Same as in Fig. 1 but for fixed $\omega_A^2=0.01\omega_{cd}^2$ and $4\pi J/cBk=0.01$ against $J/(Zen_dv_A)$.

parameters is guided by the fact that $J/(Zen_d)$ is typically the drift velocity of the plasma particles against the dust grains and since dust carries 1–2 negative charge in the noctilucent layer, assuming $n_d \sim 10\ \text{cm}^{-3}$, and Alfvén velocity $v_A \sim 10$ km/s for submicron sized grain, we see that the plasma drift velocity (3–5 km/s) is smaller than the Alfvén velocity. Thus we choose $J/(Zen_d)=0.1v_A$. The ratio of the Alfvén to the dust cyclotron frequency for given plasma parameters implies fixing the wavelength of fluctuations. We see from Fig. 1(a) (bold line) that the current driven instability grows only at long wavelength in conformity with Eq. (15). The corresponding maximum growth rate is $\sim 0.25\omega_A$, which is very small considering that here the Alfvén frequency is very small since $k \to 0$. With decreasing wavelength, the growth rate diminishes, completely disappearing at 0.5, i.e., when $\lambda \lesssim cB/4J$. The real part of the frequency (dotted curve) does not change much over the entire span.

In Fig. 1(b) the growth rate (bold curve) of the current driven instability is shown against ω_A/ω_{cd} for the fixed $J/Zen_d=0.1v_A$ and $4\pi J/cBk=0.01$. The growth rate is very similar to the previous case, except now that the wavelength at which the instability disappears is slightly larger. Note that both the whistler and the ion-cyclotron modes are the short wavelength part of the spectrum and thus it would appear that with increasing ω_A/ω_{cd}, the instability, if it is solely due to the Hall diffusion, should grow contrary to what is seen in Fig. 1(b). However, given that the necessary condition $C_0<0$ is easily satisfied for the long wavelength fluctuations, which corresponds to the Alfvén waves [please see Eq. (16)], it is not surprising that with increasing ω_A/ω_{cd}, instability disappears. Therefore, although the instability is caused by the Hall diffusion in the medium, it is the long wavelength Alfvén wave that becomes unstable. The real part of the frequency is similar to the previous case.

How is the growth rate affected by $J/(Zen_dv_A)$? As noted above, $J/(Zen_dv_A)$ is essentially the ratio of the plasma drift to the Alfvén velocity. We plot in Fig. 2 both the real and the imaginary parts of the ω/ω_A against $J/(Zen_dv_A)$ for fixed $\omega_A^2=0.01\omega_{cd}^2$ and $4\pi J/cBk=0.01$. The maximum growth rate of the instability is $0.55\omega_A$ when $J/Zen_d \sim v_A$, implying that the waves get maximum free energy when the drift J/Zen_d and the Alfvén velocities are comparable. Note that the values of the real and imaginary parts of the frequency are comparable when $J/Zen_d \sim v_A$, which means that there is an equipartition of the free energy between decaying and growing parts of the wave. Only when $J/(Zen_dv_A) > v_A$, the imaginary part of the frequency dominates the real part. The growth rate slowly decreases with increasing $J/(Zen_dv_A)$ and becomes zero when $J/(Zen_dv_A)=150$ (not shown in the figure). Since it is the long wavelength Alfvén waves that are destabilized by the Hall diffusion, it is not surprising that the maximum growth rate is achieved when $J/Zen_d \sim v_A$, i.e., plasma drift speed is comparable to the Alfvén speed. When $J/(Zen_dv_A) > 1$, the Alfvén wave will lag the plasma particle drift, and thus will be unable to get the plasma kinetic energy at the same rate as it was getting when $J/(Zen_dv_A) \sim 1$. Thus, we see the growth rate as well as the real part of the frequency decreases and will ultimately become zero for very large $J/(Zen_dv_A)$.

IV. APPLICATION AND SUMMARY

In the vicinity of bright discrete auroral arcs, field aligned currents are often observed.[30] These field aligned currents are also a source of free energy. The summer mesopause (80–90 km), which is the coldest place in the Earth's environment ($T \lesssim 100$ K), is associated with the polar mesospheric echo (PMSE)—a strong radar backscatter ($\sim$50 MHz–1.3 GHz).[33] These observations indicate that during the PMSE and noctilucent cloud formation, a large amount of dust is present in the medium.[3] Thus, present results can be applied to the Earth's lower ionosphere. The maximum growth rate of $\sim 0.5\omega_A$ at $J/Zen_d \sim v_A$ for $v_A \sim 10$ km/s implies that the instability growth rate is $\sim 5(k/\text{km})\text{s}^{-1}$, suggesting that for fluctuations of wave number $k = 10$ km^{-1}, the growth rate is $\sim$50 Hz. This growth rate compares favorably with the Farley–Buneman instability.[27–29] However, as has been noted above, the present instability is electromagnetic in nature, whereas Farley–Buneman instability is electrostatic, and thus a detailed comparison of the two instabilities is not possible.

To summarize, the current driven instability in a collisional, magnetized, dusty medium has been analyzed in the present work. The low frequency, long wavelength waves in the magnetic field aligned current medium can become unstable to the low frequency fluctuations if the ratio of the current to the background field is larger than the light speed times the wave number. The growth rate of the instability depends on the ratio of the Alfvén to the dust cyclotron frequency as well as on J/Zen_d. The growth rate of the instability compares favorably with Farley–Buneman instability, although the physical mechanism of the present instability is very different from the Farley–Buneman instability.

ACKNOWLEDGMENTS

The support of the Australian Research Council is gratefully acknowledged for the present work.

[1] L. Spitzer, *Physical Processes in the Interstellar Medium* (Wiley, New York, 1978).
[2] C. J. Mitchell, M. Horanyi, O. Havnes, and C. C. Porco, Science **311**, 1587 (2006).
[3] O. Havnes, J. Troim, T. Blix, W. Mortensen, L. I. Naesheim, E. Thrane, and T. Tonnesen, J. Geophys. Res. **101**, 10839, doi:10.1029/96JA00003 (1996).
[4] S. V. Vladimirov, K. Ostrikov, and A. A. Samarian, *Physics and Applications of Complex Plasmas* (Imperial College Press, London, 2005).
[5] W. A. Scales and R. Ganguli, New J. Phys. **6**, 12 (2004).
[6] W. A. Scales and C. Chen, Adv. Space Res. **41**, 50 (2008).
[7] B. P. Pandey, K. Avinash, and C. B. Dwivedi, Phys. Rev. E **49**, 5599 (1994).
[8] B. P. Pandey, G. S. Lakhina, and V. Krishan, Phys. Rev. E **60**, 7412 (1999).
[9] S. V. Vladimirov and K. Ostrikov, Phys. Rep. **393**, 175 (2004).
[10] B. P. Pandey, A. A. Samarian, and S. V. Vladimirov, Phys. Plasmas **14**, 093703 (2007).
[11] J. R. Bhatt and B. P. Pandey, Phys. Rev. E **50**, 3980 (1994).
[12] B. P. Pandey and C. B. Dwivedi, Pramana, J. Phys. **45**, 255 (1995).
[13] E. C. Whipple, Rep. Prog. Phys. **44**, 1197 (1981).
[14] B. P. Pandey and S. V. Vladimirov, Phys. Plasmas **14**, 052105 (2007).
[15] B. P. Pandey and J. Vranjes, Phys. Plasmas **13**, 122106 (2006).
[16] W. Pilipp, T. W. Hartquist, O. Havnes, and G. E. Morfill, Astrophys. J. **314**, 341 (1987).
[17] O. Havnes, T. W. Hartquist, and W. Pilipp, Astron. Astrophys. **217**, L13 (1989).
[18] N. F. Cramer and S. V. Vladimirov, Phys. Scr. **53**, 586 (1996).
[19] M. C. de Juli, D. Falceta-Gonzalves, and V. Jatenco-Pereira, Adv. Space Res. **35**, 925 (2005).
[20] P. K. Shukla, I. Kourakis, and L. Stenflo, Phys. Plasmas **12**, 024501 (2005).
[21] B. P. Pandey and M. Wardle, Mon. Not. R. Astron. Soc. **385**, 2269 (2008).
[22] D. T. Farley, J. Geophys. Res. **68**, 401, doi:10.1029/JZ068i002p00401 (1963).
[23] O. Buneman, Phys. Rev. Lett. **10**, 285 (1963).
[24] M. J. Schmidt and P. S. Gary, J. Geophys. Res. **78**, 8261, doi:10.1029/JA078i034p08261 (1973).
[25] S. L. Ossakow, K. Papadopoulos, K. Orens, and T. Coffey, J. Geophys. Res. **80**, 141, doi:10.1029/JA080i001p00141 (1975).
[26] M. C. Kelly, *The Earth's Ionosphere: Plasma Physics and Electrodynamics* (Academic, New York, 1989).
[27] N. D'Angelo, Planet. Space Sci. **51**, 393 (2003).
[28] M. Rosenberg and P. K. Shukla, J. Geophys. Res. **105**, 23135, doi:10.1029/2000JA900080 (2000).
[29] W. A. Scales, Phys. Scr. **T107**, 107 (2004).
[30] T. N. Davis, Space Sci. Rev. **22**, 77 (1978).
[31] N. D'Angelo, Phys. Lett. A **292**, 195 (2001).
[32] P. Muralikrishna and V. H. Kulkarni, Ann. Geophys. **24**, 2949 (2006).
[33] J. Y. N. Cho and M. C. Kelly, Rev. Geophys. **31**, 243, doi:10.1029/93RG01535 (1993).