Information Propagation Speed in Delay Tolerant Networks: Analytic Upper Bounds

Philippe Jacquet
INRIA
Rocquencourt, France
philippe.jacquet@inria.fr

Bernard Mans
Macquarie University
Sydney, Australia
bmans@ics.mq.edu.au

Georgios Rodolakis
Macquarie University
Sydney Australia
georgios@ics.mq.edu.au

***Abstract*—Delay/Disruption Tolerant Networks (DTNs) or Intermittently Connected Mobile Networks (ICNs) are mobile ad hoc networks where end-to-end multi-hop paths may not exist and communication routes may only be available through time and mobility. While most of the research is dedicated to the design of routing protocols, very few properties of such networks are known. In a recent paper [6], the authors provided analytical upper bounds for the information propagation speed in DTNs when they are modeled as two-dimensional Unit Disk Graphs.**

In this paper, we extend this study to other models by using analytical tools to derive theoretical upper-bounds of the information propagation speed. Firstly, we will present results for DTNs mapped in a space of dimension D, where D varies from 1 to 3. Secondly, we will depart from the Unit Disk Graph model to consider a more realistic model where a node captures a packet sent at distance r with probability $p(r)$.

I. Introduction

Delay/Disruption Tolerant Networks (DTNs) or Intermittently Connected Mobile Networks (ICNs) are mobile ad hoc networks where end-to-end multi-hop paths may not exist and communication routes may only be available through time and mobility (see [8] for a survey of routing protocols). Our objective is to evaluate the maximum speed at which a piece of information can propagate (when the mobile network is temporarily disconnected, the information propagation is stalled as long as the node mobility does not allow the information to jump to different connected components of the network). The packet is either transmitted or carried by a node, requiring a *store-carry-and-forward* routing model. Thus, a "path" is an alternation of packet transmissions and packet carriages that connects a source to a destination, and is better referred (from now on) as a **journey**.

Informally, we aim to find the "foremost" journey (in time) that connects a source to a destination. As we are interested in the overall propagation speed, between any source and any destination, we will denote it as the "shortest" journey without lack of generality. When the node mobility is known in advance (*e.g.,* for fixed schedule dynamic networks such as satellite networks), the shortest journey in time can be calculated in polynomial time (*e.g.,* [3]).

Our aim is to compute the fastest possible information propagation without predictive knowledge. In [6], the authors have proved theoretical upper bounds when the network is modeled as a Unit Disk Graph (UDG), and when the high rate of transmissions does not affect the range of transmission (conversely to known models such as Gupta and Kumar [4]).

In this paper, we will use analytical tools to extend the theoretical upper bounds of [6] to more realistic models of networks. We will first recall the model and results of [6]. We will show that the methodology can be extended to the case of networks when they are mapped in a space of dimension 1 and 3. We will adopt a didactic approach. In Section III, we will initially study a simpler problem where nodes are only allowed to transmit when they change direction or when they receive the beacon. In Section IV, we will generalize the approach by allowing nodes to transmit only when they have a new neighbor or when they receive a new information (*i.e.,* when a local event occurs). Finally, in Section V, we will consider a more realistic model where a node captures a packet sent at distance r with probability $p(r)$.

II. Network Modeled as a Multi-Dimensional Unit-Disk Graph

In this section, we consider the problems when the network map is in a space of dimension D from $D = 1$ to $D = 3$. The network in our model is infinite, therefore it is surely disconnected. This generalizes the case $D = 2$ treated throughout in [6]: an infinite two-dimensional network with uniform node distribution and with constant density ν. Two nodes at distance smaller than one can exchange information. The average number of neighbors per node is therefore $\pi\nu$. We assume that each node follows an independent walk of speed v. The node keeps a uniform speed but changes direction at Poisson rate τ. When $\tau \to \infty$ we are on the Brownian limit; when $\tau \to 0$, we are on a random way point-like model. The motion direction angles are isotropic (*e.g.,* for $D = 2$, they are uniformly distributed between 0 and 2π).

In [6], the fastest possible information propagation is computed by using an algorithm that contains all possible "shortest" journeys: the full broadcast. The information is called the *beacon*. Every time a new node is in range of a node which carries a copy of the beacon, the latter node transmits another copy of the beacon to the new node. Transmission is done indifferently by broadcast or unicast, since this does not change the details of our model. Again, the facts that the high rate of transmission will affect the range of transmission (*e.g.,* Gupta and Kumar [4]) and that packet losses will occur, are not considered. The model is intended to unfold all journeys from a source to a destination for which a window of opportunity for transmission may exist.

Our result is the evaluation of an upper bound of the information propagation speed (concerning all possible routing

978-1-4244-2571-6/08/$25.00 ©2008 IEEE

strategies). In fact, the concept of propagation speed is probabilistic. Let us consider a mobile at coordinate $\mathbf{z} = (x, y)$ at time $t = 0$; $q(\mathbf{z}, t)$ denotes the probability that the mobile receives the beacon before time t.

A scalar $\sigma_0 > 0$ is an upper bound of the propagation speed, if for all $\sigma > \sigma_0 \lim q(\mathbf{z}, \frac{|\mathbf{z}|}{\sigma}) = 0$ when $|\mathbf{z}| \to \infty$. For example if $q(\mathbf{z}, t) < \exp(-a|\mathbf{z}| + bt + c)$ then quantity $\frac{b}{a}$ is an upper bound on propagation speed.

To derive the upper bound for the speed, we use an asymptotic analysis based on the Saddle Point method on the Laplace transform of the journey probability density function.

Theorem 1: An upper bound of the information propagation speed in a network (with node radio range R, node density ν, node speed v, node direction change rate τ, in a space of dimension D) is the smallest ratio $\frac{\theta}{\rho}$ attained by the elements (ρ, θ) of the set $\mathcal{K}$ made of the non negative tuples (ρ, θ) that are root of

$$\frac{1}{Z_D(\rho, \theta)} - \tau - G_D(\rho R),$$

with

$$G_D(\rho) = \frac{f_D R^{D-1} \Xi_D(\rho)}{1 - V_D R^D \nu \Psi_D(\rho)},$$

where the values of $Z_D(\rho, \theta)$, $\Xi_D(\rho)$, $\Psi_D(\rho)$, V_D and f_D are defined (depending on D) in Table I and where functions $I_0()$ and $I_1()$ are the *modified Bessel functions* (see [1]).

The main result of [6] can now be described as a corollary but still provides the same bound.

Remark: Quantity ρ is expressed as an inverse of distance and quantity θ is expressed as an inverse of time. Therefore, the ratio $\frac{\theta}{\rho}$ is the dimension of a speed. In the remaining of the paper, and *w.l.o.g.*, we will assume that $R = 1$.

Since quantities $\Xi_D(x)$ and $\Psi_D(x)$ are both greater than 1, the previous expression has meaning when $\nu < \frac{1}{V_D}$ where V_D is the volume of the unit hyper-sphere. When $\nu \geq \frac{1}{V_D}$, our model indicates an infinite propagation speed. Therefore our model is interesting when ν is small. Similarly, when $v = 0$ and $\nu < \frac{1}{V_D}$, the propagation speed is zero.

Corollary 1: Let $v > 0$ and $\tau > 0$, when $\nu \to 0$, the propagation speed is asymptotically equivalent to $v\sqrt{\frac{2f_D}{\tau}}$.

It is important to notice that the speed diminishes with the square root of the density ν. However this estimate does not hold in the case $\tau = 0$ that we will fully depict as the random waypoint model limit later in the paper.

Corollary 2: In the random waypoint limit, *i.e.,* when $\tau = 0$, the propagation speed upper bound is $(1 + O(\nu^2))v$.

It turns out that the propagation speed upper-bound at the limit is v. This is rather surprising, because we would expect that the propagation speed would tend to zero when $\nu \to 0$. The explanation is that we first set ν and then look at the propagation speed when the mobile nodes are located at location infinitely far from the beaconing source. This is different from considering first a node at a remote location $\mathbf{z}$ from the source and then let $\nu \to 0$. In fact we have $q(\mathbf{z}, t) < \exp(-a|\mathbf{z}| + bt + c)$ and $c \to \infty$ when $\nu \to 0$ confirming that propagation speed tends to zero when $\nu \to 0$ when $\mathbf{z}$ is fixed, but tends to $(1+O(\nu^2))v$ when ν is fixed and $|\mathbf{z}| \to \infty$. A rule of thumb for having v lower-bounding the propagation speed is that as soon as the set of attained nodes is large enough there is likely to be one heading toward $\mathbf{z}$.

III. Simplified Approach

In this section we suppose that nodes can only transmit when they change direction or when they receive the beacon.

A. Journey Analysis

Our analysis is based on journey segmentation between the source and the destination. Formally, a journey is a space-time trajectory of the beacon between the source and a destination. We assume that time zero is when the source transmits, and we will check at what time t, the beacon is emitted at distance smaller than one to the destination at coordinate $\mathbf{z} = (x, y)$. The beacon can take many journeys in parallel, due to the broadcast nature of radio transmission, and the fact that the beacon stays in the memory of the emitter (and therefore can be emitted several times in the trajectory of a mobile node). In a first approach, and in order to simplify, we will assume that the destination is fixed (the node does not move). We will later modify the model in order to support the destination motion.

We will consider only simple journeys, *i.e.,* journeys which never return twice through the same node. This restriction does not affect the analysis, since if a journey arrives to the destination at time t, then we can extract a simple journey from this journey which arrives at time t too.

To simplify the presentation we now consider journeys as if they were enumerable objects and can be affected to a probability weight. In the following we will consider a journey as a discrete event in a continuous set, therefore the probability weight should be converted into a probability density.

Let $\mathcal{C}$ be a simple journey. Let $Z(\mathcal{C})$ be the terminal point. Let $T(\mathcal{C})$ be the time at which the journey terminates. Let $p(\mathcal{C})$ be the probability of journey $\mathcal{C}$.

We call $p(\mathbf{z}, t)$ the average density of journeys that arrive at $\mathbf{z}$ before time t:

$$p(\mathbf{z}, t) = \lim_{r \to \infty} \frac{1}{\pi r^2} \sum_{|\mathbf{z} - Z(\mathcal{C})| < r, T(\mathcal{C}) < t} p(\mathcal{C}) \ .$$

1) Journey Segmentation: In the following, we split the journey into segments $\mathcal{C} = (s_1, s_2, \ldots, s_k)$, such that $p(\mathcal{C}) = p(s_1)p(s_2)\cdots p(s_k)$.

Therefore, a journey is made of two kinds of segments:

- Carry segments s_c: the beacon is held until the next change of direction;
- Emission segments s_e: the beacon is transmitted to a neighbor.

In dimension 2, a carry segment s_c is a space-time vector $(tv\cos\phi, tv\sin\phi, t)$ where ϕ is the direction angle of the motion vector which belongs to $[0, 2\pi)$, and t is the time duration of segment. At this level of the analysis the segments are no longer enumerable and we refer to the probability density of segments, which is

$$p(s_c) = \frac{1}{2\pi} \tau e^{-\tau t},$$

since all angles have the same probability.

D	$Z_D(\rho,\theta)$	$\Xi_D(\rho)$	$\Psi_D(\rho)$	V_D	f_D
1 **[this paper]**	$\frac{\tau+\theta}{(\tau+\theta)^2-\rho^2 v^2}$	$\cosh(\rho)$	$\frac{\sinh(\rho)}{\rho}$	2	$v\nu$
2 [6]	$\frac{1}{\sqrt{(\tau+\theta)^2-\rho^2 v^2}}$	$I_0(\rho)$	$\frac{2}{\rho}I_1(\rho)$	π	$\frac{8v}{\pi}\nu$
3 **[this paper]**	$\frac{1}{2\rho v}\log\left(\frac{\tau+\theta+\rho v}{\tau+\theta-\rho v}\right)$	$\frac{\sinh(\rho)}{\rho}$	$\frac{3}{\rho^3}(\rho\cosh(\rho)-\sinh(\rho))$	$\frac{4}{3}\pi$	$\frac{4}{3}\pi v\nu$

TABLE I
$Z_D(\rho,\theta)$, $\Xi_D(\rho)$, $\Psi_D(\rho)$, V_D AND f_D ARE DEFINED (DEPENDING ON D).

An emission segment s_e is a space time vector $(r\cos\phi, r\sin\phi, \epsilon)$ with $r \in [0,1)$. The quantity ϵ is the time needed for a transmission, which is assumed small compared to typical node mobility, hence practically $\epsilon = 0$. W.l.o.g, we take later $\epsilon = 0$ in order to insist on the fact that transmission times are infinitely smaller than moving times. With respect to variables ϕ and r, we have the density:

$$p(s_e) = \nu r \ .$$

Similarly, in dimension D, carry/emission segments are space time vectors of dimension $D+1$. Notice that a journey of k segments is made of k space-time vectors, therefore $p(\mathcal{C})$ is a probability density of a vector of dimension $(D+1)k$.

2) Journey Laplace Transform: Let ζ be a space vector and θ a scalar. We denote $w_D(\zeta,\theta)$ the journey Laplace transform defined by

$$\begin{aligned} w_D(\zeta,\theta) &= E(\exp(-\zeta.Z(\mathcal{C}) - \theta T(\mathcal{C}))) \\ &= \textstyle\sum_{\mathcal{C}} p(\mathcal{C})\exp(-\zeta.Z(\mathcal{C}) - \theta T(\mathcal{C})) \end{aligned}$$

defined for a domain definition for (ζ,θ). Notice that $\zeta.Z(\mathcal{C})$ is the dot product of two space vectors.

By virtue of the inverse Laplace transform of $w_D(\zeta,\theta)$, we obtain quantity $p(\mathbf{z},t)$, which is the average density of journeys that arrive at $\mathbf{z}$ before time t. For example, in dimension 2, we have:

$$p(\mathbf{z},t) = (\frac{1}{2i\pi})^3 \int\int w_2(\zeta,\theta)e^{\zeta.\mathbf{z}+t\theta}d\zeta\frac{d\theta}{\theta}, \qquad (1)$$

where the integration domains are plans parallel to the imaginary plan in the definition domain.

A journey being an arbitrary sequence of carry segments and emission segments, we have the following simple expression inspired from combinatorial analysis[1]:

$$w_D(\zeta,\theta) = \frac{1}{1-\sum_{s_c} p(s_c)e^{-\langle s_c,(\zeta,\theta)\rangle} - \sum_{s_e} p(s_e)e^{-\langle s_e,(\zeta,\theta)\rangle}}, \qquad (2)$$

where the quantity $\langle s_c, (\zeta,\theta)\rangle$ is the dot product between two space-time vectors.

In dimension 2, we have the expression:

$$\begin{aligned} \textstyle\sum_{s_c} p(s_c)e^{-\langle s_c,(\zeta,\theta)\rangle} &= \int_0^{2\pi}\frac{d\phi}{2\pi}\int_0^\infty \tau e^{|\zeta|vt\cos\phi}e^{-\theta t}e^{-\tau t}dt \\ &= \frac{\tau}{\sqrt{(\tau+\theta)^2-|\zeta|^2v^2}}. \end{aligned}$$

Similarly,

$$\begin{aligned} \textstyle\sum_{s_e} p(s_e)e^{-\langle s_e,(\zeta,\theta)\rangle} &= \int_0^{2\pi} d\phi\int_0^1 \nu r dr e^{r|\zeta|\cos\phi} \\ &= \pi\nu\Psi_2(|\zeta|), \end{aligned}$$

[1]This is the equivalent of the formal identity $\frac{1}{1-y} = 1+y+y^2+y^3+\ldots$, which represents the Laplace transform of an arbitrary sequence of random variables with Laplace transform y.

with $\Psi_2(x) = \sum_{k\geq 0}(\frac{x}{2})^{2k}\frac{1}{(k+1)!k!} = \frac{2}{x}I_1(x)$, where $I_1(x)$ is a modified Bessel function.

Equivalently, by integrating in dimension D, we have:

$$\begin{aligned} E(e^{-\langle(\zeta,\theta),s_c\rangle}) &= \tau Z_D(|\zeta|,\theta), \\ E(e^{-\langle(\zeta,\theta),s_e\rangle}) &= \nu V_D\Psi_D(|\zeta|), \end{aligned}$$

where functions $Z_D()$, $\Psi_D()$ and V_D are defined in Table I.

B. Saddle Point Analysis

The journey Laplace transform is in denominator of the form $1 - D(|\zeta|,\theta)$, with $D(|\zeta|,\theta) = E(e^{-\langle(\zeta,\theta),s_c\rangle}) + E(e^{-\langle(\zeta,\theta),s_e\rangle})$. The key of the analysis is the set $\mathcal{K}$ of pairs (ρ,θ) such that $D(\rho,\theta) = 1$, called the *Kernel*. A function w_D of this form implies that $p(\mathbf{z},t) = O(\exp(-\rho_0|\mathbf{z}| + \theta_0 t))$, where (ρ_0,θ_0) is the element of $\mathcal{K}$ that minimizes $-\rho|\mathbf{z}|+\theta t$. Due to space restrictions and for simplicity, we only detail the case of dimension 2.

Theorem 2: (From [6].) When $|\mathbf{z}|$ and t tend both to infinity we have:

$$p(\mathbf{z},t) = (1+O(\frac{1}{\sqrt{t}}))\frac{\exp(-\rho_0|\mathbf{z}|+\theta_0 t)}{2\pi\theta_0\sqrt{\frac{D_\theta D_\rho}{\rho_0}\nabla_2 D(t,|\mathbf{z}|)}},$$

where (ρ_0,θ_0) is the element of $\mathcal{K}$ that minimizes $-\rho|\mathbf{z}|+\theta t$. We have $D_\rho = \frac{\partial}{\partial\rho}D$, $D_\theta = \frac{\partial}{\partial\theta}D$ and $\nabla_2 D(x,y) = x^2\frac{\partial^2 D}{\partial\rho^2} + y^2\frac{\partial^2 D}{\partial\theta^2} + 2xy\frac{\partial^2 D}{\partial\rho\partial\theta}$

Proof: Equation (1) for $p(\mathbf{z},t)$ can be rewritten as

$$(\frac{1}{2i\pi})^3\int\int w_2(\zeta_0+i\theta,\theta_0+i\theta)e^{\langle(\zeta_0,\theta_0),(\mathbf{z},t)\rangle+i\langle(\zeta,\theta),(\mathbf{z},t)\rangle}d\zeta\frac{d\theta}{\theta_0+i\theta}, \qquad (3)$$

where the integration domain consists of real planes.

The function $1 - D(\rho,\theta)$ has a definition domain $\{(\rho,\theta), \Re((\theta+\tau)^2 - \rho^2v^2) > 0\}$ and, when θ varies, it has a simple root at

$$\theta(\rho) = \sqrt{(\frac{\tau}{1-\pi\nu\Psi_2(\rho)})^2 + \rho^2v^2} - \tau.$$

Notice that $(\rho,\theta(\rho))$ describes the set $\mathcal{K}$. In order to have $\rho > 0$ for some elements in $\mathcal{K}$ and to apply a consistent analysis we need the condition $\pi\nu < 1$.

Therefore the residues analysis gives:

$$p(|\mathbf{z}|,t) = I(\mathbf{z},t) + R(\mathbf{z},t)$$

where

$$I(\mathbf{z},t) = \frac{1}{(2i\pi)^2}\int\frac{\exp\langle(\zeta,\theta(|\zeta|)),(\mathbf{z},t)\rangle}{\theta(\rho)D_\theta(\rho,\theta(\rho))}d\zeta$$

with $D_\theta = \frac{\partial}{\partial\rho}D$, and $R(\mathbf{z},t)$ is the integral of $\frac{\exp(\langle(\zeta,\theta),(\mathbf{z},t)\rangle}{(1-D(|\zeta|,\theta)\theta}$ when $\rho v - \tau < \Re(\theta) < \theta(\rho)$.

We will show at the end that $R(\mathbf{z},t) = O(e^{-Bt}I(\mathbf{z},t))$ for some $B > 0$.

We first focus on $I(\mathbf{z},t)$ by using saddle point techniques. Let ζ_0 be the value that minimizes $(\zeta, \mathbf{z} + \theta(|\zeta|)t)$. Obviously $\zeta_0 = -\frac{\rho_0}{|\mathbf{z}|}\mathbf{z}$ with ρ_0 that minimizes $-\rho|\mathbf{z}| + \theta(\rho)t$.

Let θ' and θ'' be the first and second derivatives of $\theta(\rho)$ respectively. We already know that $\theta' = \frac{|\mathbf{z}|}{t}$. Since $D(\rho, \theta(\rho)) = 1$, by derivation with respect to ρ we have $D_\rho + D_\theta \theta' = 0$ and, by second derivation, $\nabla_2 D(1, \theta') + D_\theta \theta'' = 0$ at $\rho = \rho_0$. Without loss of generality we assume that $\mathbf{z} = (-|\mathbf{z}|, 0)$ and $\zeta = (-\rho_0, 0) + (x, y)$, thus

$$|\zeta| = \rho_0 - x + \frac{1}{2\rho_0}y^2 + O(x^3 + y^3),$$

and in turn

$$\zeta.\mathbf{z} + \theta(|\zeta|)t = -\rho_0|\mathbf{z}| + \theta_0 t + \frac{t}{2}(\theta'' x^2 + \frac{\theta'}{\rho_0}y^2) + O((x^3 + y^3)t).$$

By change of variable $\zeta = (-\rho_0, 0) + \frac{i}{\sqrt{t}}(x, y)$, we get

$$\begin{aligned} \zeta.\mathbf{z} + \theta(|\zeta|)t &= -\rho_0|\mathbf{z}| + \theta_0 t - \tfrac{1}{2}(\theta'' x^2 + \tfrac{\theta'}{\rho_0}y^2) \\ &\quad + O((x^3 + y^3)t^{-1/2}) \end{aligned} \tag{4}$$

$$\begin{aligned} I(\mathbf{z}, t) &= \frac{\exp(-\rho_0|\mathbf{z}| + \theta_0 t)}{(2\pi)^2} \int\int \frac{\exp(\frac{1}{2}(\theta'' x^2 + \frac{\theta'}{\rho_0}y^2))}{\theta D_\theta} dx dy \\ &\quad \times (1 + O(t^{-1/2})) \\ &= \frac{\exp(-\rho_0|\mathbf{z}| + \theta_0 t)}{2\pi\theta_0 D_\theta \sqrt{\frac{\theta'\theta''}{\rho_0}}}(1 + O(t^{-1/2})) \ . \end{aligned} \tag{5}$$

To terminate with integral $R(\mathbf{z}, t)$, we identify

$$B = \theta_0 - \rho_0 v + \tau.$$

■

C. Information Propagation Speed

Let $q(\mathbf{z}, t)$ be the probability that there exists a journey that arrives at distance less than 1 to $\mathbf{z}$ before time t.

Theorem 3: (From [6].) We have the upper-bound:

$$q(\mathbf{z}, t) \leq \int_{|\mathbf{z} - \mathbf{z}'| < 1} p(\mathbf{z}', t) d\mathbf{z}' \ .$$

Therefore, $q(\mathbf{z}, t) = O(p(\mathbf{z}, t))$. Following the saddle point analysis, $q(\mathbf{z}, t)$ vanishes very quickly when t is smaller than the value such that $-\rho_0|\mathbf{z}| + \theta_0 t = 0$, *i.e.*, when $\frac{\theta_0}{\rho_0} = \frac{z}{t} = \theta'(\rho_0)$. This ratio gives the upper-bound of the propagation speed. In other words, point (ρ_0, θ_0) achieves the lowest ratio $\frac{\theta}{\rho}$ in the kernel set $\mathcal{K}$.

D. Moving Destination

In the previous evaluation, we assumed that the destination does not move during the propagation of the information. Now we consider that the destination can move as the other nodes, starting at position $\mathbf{z}$ at time $t = 0$.

Theorem 4: When the destination moves as the other nodes in the network, then the asymptotic propagation speed upper bound does not change when $(\mathbf{z}, t)$ tend to infinity.

For this end, it suffices to multiply the journey Laplace transform $w_D(\zeta, \theta)$ with the Laplace transform of the node excursion from its original position. The excursion Laplace transform is obtained from the carry segment Laplace transform, and it has the expression $\frac{Z_D(|\zeta|, \theta)}{1 - \tau Z_D(|\zeta|, \theta)}$ where $Z_D()$ is defined in Table I. The new Laplace transform has two sets of poles, the set $\mathcal{K}$ and the set $\mathcal{K}'$ corresponding to the set $\{(\rho, \theta) \, : \, \theta = \rho v - \tau\}$. The last set is dominated on the right by $\mathcal{K}$, for all $(\rho, \theta') \in \mathcal{K}'$, there is a (ρ, θ) in $\mathcal{K}$ with $\theta > \theta' + B$. Therefore the contributions from $\mathcal{K}'$ will be exponentially negligible (of order $\exp(-Bt)$) compared to the main contribution from $\mathcal{K}$, and the propagation speed upper-bound does not change from the value computed in the previous subsection.

Namely, in dimension 2, the main contribution from $\mathcal{K}$ gives

$$p(\mathbf{z}, t) \sim \frac{\exp(-\rho_0|\mathbf{z}| + \theta_0 t)}{(\frac{1}{Z_D(|\mathbf{z}|, \theta)} - \tau) 2\pi\theta_0 \sqrt{\frac{D_\theta D_\rho}{\rho_0}\nabla_2 D(t, |\mathbf{z}|)}}.$$

IV. Realistic Approach (Proof of Theorem 1)

In this approach, we assume that the nodes can transmit only when they meet new neighbors or when they receive the beacon. According to [5], when nodes move at speed v with isotropic direction in a 2-D domain, the frequency f_2 at which new neighbors appear satisfies $f_2 = \frac{8v}{\pi}\nu$. In Table I, we have evaluated the frequency f_D in case of multi-dimensional unit disk graphs.

Lemma 1: The journey Laplace transform with realistic approach has the expression:

$$w_D(\zeta, \theta) = \frac{Z_D(|\zeta|, \theta)}{1 - Z_D(|\zeta|, \theta)(\tau + G_D(|\zeta|))}$$

with

$$G_D(\rho) = \frac{f_D \Xi_D(\rho)}{1 - V_D \nu \Psi_D(\rho)}$$

where the values of $Z_D(\rho, \theta)$, $\Xi_D(\rho)$, $\Psi_D(\rho)$, V_D and f_D are defined (depending on D) in Table I.

Proof: A typical journey is an arbitrary mixture of carry segments s_c (the beacon is held until the next change of direction) and carry-and-transmit segments s_{ce} (the beacon is carried, then transmitted to a new neighbor and broadcasted in the connected component). A carry-and-transmit segment is made of a carry-to-neighbor segment s_{cn}, and a transmit-to-neighbor segment s_{tn} followed by an arbitrary number of emission segments s_e.

In dimension 2, the carry-to-neighbor segment is a space-time vector $(v\cos\phi, v\sin\phi, t)$ where ϕ is a number in $[0, 2\pi)$ and t is a non negative number. We have the density

$$p(s_{cn}) = e^{-\tau t}\frac{f_2}{2\pi},$$

with Laplace transform :

$$E(e^{\langle(\zeta, \theta), s_{cn}\rangle}) = \frac{f_2}{\sqrt{(\theta + \tau)^2 - \rho^2 v^2}}$$

For the transmit-to-neighbor segment we assume that the beacon is transmitted to the new neighbor only at distance 1, since the other neighbors having already received the beacon. Therefore, a transmit-to-neighbor segment is a space-time vector $(\cos\phi, \sin\phi, 0)$, and $p(s_{tn}) = \frac{1}{2\pi}$.

Computing the Laplace transform yields:

$$\begin{array}{rcl} E(e^{\langle(\zeta,\theta),s_{tn}\rangle}) & = & \int_0^{2\pi} e^{|\zeta|\cos\phi}\frac{d\phi}{2\pi} \\ & = & I_0(|\zeta|). \end{array}$$

Finally, the Laplace transform of the carry-and-transmit segment is

$$\frac{f_2 I_0(\rho)}{\sqrt{(\theta+\tau)^2-\rho^2v^2}(1-\pi\nu\Psi_2(\rho))}.$$

Similarly, in dimension D, we have the Laplace transforms:

$$\begin{array}{rcl} E(e^{-\langle(\zeta,\theta),s_{cn}\rangle}) & = & f_D Z_D(|\zeta|,\theta), \\ E(e^{-\langle(\zeta,\theta),s_{tn}\rangle}) & = & \Xi_D(|\zeta|), \end{array}$$

and finally:

$$E(e^{-\langle(\zeta,\theta),s_{ce}\rangle}) = \frac{f_D Z_D(|\zeta|,\theta)\Xi_D(|\zeta|)}{1-V_D\nu\Psi_D(|\zeta|)} = Z_D(|\zeta|,\theta)G_D(|\zeta|),$$

where the values of $Z_D()$, $\Xi_D()$, and f_D are defined (depending on D) in Table I.

Therefore, the function $w_D(\zeta,\theta)$ is in denominator $1-E(e^{-\langle(\zeta,\theta),s_c\rangle})-E(e^{-\langle(\zeta,\theta),s_{ce}\rangle})$. In the numerator, we have the function $Z_D(|\zeta|,\theta)$, *i.e.,* the excursion Laplace transform which corresponds to the last straight line before the destination. This terminates the proof. ■

This leads to the proof of Theorem 1.

Proof: The Kernel $\mathcal{K}$ of $w_D(\zeta,\theta)$ is the root of the denominator:

$$1-Z_D(\rho,\theta)\left(\tau+\frac{f_D\Xi_D(\rho)}{1-V_D\nu\Psi_D(\rho)}\right).$$

Therefore, following the saddle point asymptotic analysis of the average number of journeys, the propagation speed upper-bound is given by the minimum ratio $\frac{\theta}{\rho}$ of (θ,ρ) in the set $\mathcal{K}$. ■

V. General Neighbor conditions

In this section ,we investigate the more realistic approach when the connectivity conditions depart from the unit disk model. We consider that a node at distance r receives a packet with probability $p(r)$ which is a given decreasing function. For example, the simultaneous transmissions in the vicinity of the transmitter or of the receiver can randomly affect the reception of the packet (see [2] for an expression of $p(r)$). Furthermore we assume that the function $p(r)$ decays super-exponentially (faster than any exponential) when r increases. This a realistic assumption, for instance in dimension 2 (see [2]), with an attenuation coefficient of 4 we already get $p(r)=1-\text{erf}(\pi^{\frac{3}{2}}\frac{Cr^2}{2})$ which decays in $\exp(-\pi^3\frac{Cr^4}{4})$, where C is a constant which depends on the traffic intensity and the desired signal to noise ratio.

We investigate a fast change model, *i.e.* a model where any pair of nodes frequently changes its neighbor status depending on its distance r. We denote $u_0(r)$ the rate at which they become neighbor when they are in non-neighbor state, and $u_1(r)$, the rate at which they become non-neighbor when they are in neighbor state. If the distance between the two nodes does not change we have $p(r)=\frac{u_0(r)}{u_0(r)+u_1(r)}$. Following [2], we can set $u_0(r)=hp(r)$ and $u_1(r)=h-hp(r)$, with h being the packet transmission rate. We assume that the changes are frequent so that nodes are always in stationary neighbor state. Otherwise the neighbor status may depend on the past history of the nodes. This assumption is important because it makes the model different of the previous model even if we would consider $p(r)=Y(1-r)$ (the Heavyside function).

In this case we obtain the following theorem in dimension 2.

Theorem 5: The journey Laplace transform has the expression:

$$w(\zeta,\theta)=\frac{1}{\sqrt{(\tau+\theta)^2-|\zeta|^2v^2}-\tau-\frac{\Xi_p(|\zeta|)}{1-\Psi_p(|\zeta|)}}$$

with

$$\begin{array}{rcl} \Xi_p(\rho) & = & \int_0^{2\pi} d\phi \int \frac{u_0(r)u_1(r)}{u_0(r)+u_1(r)} e^{\rho r\cos\phi} r dr\nu \\ & = & 2\pi\nu \int I_0(\rho r)\frac{u_0(r)u_1(r)}{u_0(r)+u_1(r)} r dr \end{array}$$

and

$$\begin{array}{rcl} \Psi_p(\rho) & = & \int_0^{2\pi} d\phi \int \frac{u_0(r)}{u_0(r)+u_1(r)} e^{\rho r\cos\phi} r dr\nu \\ & = & 2\pi\nu \int I_0(\rho r)\frac{u_0(r)}{u_0(r)+u_1(r)} r dr. \end{array}$$

Proof: The main change is in the fact that the rate at which a node at distance r pops up as a new neighbor is upper bounded by $\frac{u_0(r)u_1(r)}{u_0(r)+u_1(r)}=(1-p(r))u_0(r)$ which leads to the expression of $\Xi_p(\rho)$. Notice that the expression does not depend on the node speed anymore since the neighbor status change rate is a function of distance and is assumed to be larger than distance change rates.

The new expression of $\Psi_p(\rho)$ is a direct consequence that any nodes at distance r can receive the beacon depending on its current status. Notice that $\Psi_p(\rho)=2\pi\nu\int I_0(\rho r)p(r)rdr$. ■

The last but not least point to be pointed out is also the fact that now

$$q(\mathbf{z},t)=\int_{\mathbf{z}'} p(|\mathbf{z}-\mathbf{z}'|)p(\mathbf{z}',t)d\mathbf{z}'$$

Using the fact that $p(r)$ decays super-exponentially we have

$$q(\mathbf{z},t)\leq p(\mathbf{z},t)\int p(r)e^{\rho_0 r}2\pi r dr = O(p(\mathbf{z},t)).$$

which allows us to use the same derivations for the information propagation speed.

References

[1] M. Abramowitz and I.A. Stegun, *Chapter 9*, Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, New York, Dover Publications, 1965.

[2] C. Adjih, E. Baccelli, T. Clausen, P. Jacquet, and G. Rodolakis, *Fish eye OLSR Scaling Properties*, IEEE Journal of Communication and Networks, vol. 6, pp. 343-351, 2004.

[3] B. Bui Xuan, A. Ferreira and A. Jarry, *Computing Shortest, Fastest, and Foremost Journeys in Dynamic Networks*, Int. Journal of Foundations of Computer Science, vol. 14(2), pp. 267-285, 2003.

[4] P. Gupta and P.R. Kumar, *The capacity of wireless networks*, IEEE Trans. on Info. Theory, vol. IT-46(2), pp. 388-404, 2000.

[5] P. Jacquet, "Control of Mobile Ad Hoc Networks", IEEE Information Theory Workshop, Uruguay, pp. 97-101, 2006.

[6] P. Jacquet and B. Mans, "Information Propagation Speed in Mobile and Delay Tolerant Networks", INRIA-RR-6390, December 2007.

[7] X. Zhang, J. Kurose, B. Levine, D. Towsley and H. Zhang, "Study of a Bus-Based Disruption Tolerant Network: Mobility Modeling and Impact on Routing", ACM MobiCom, pp. 195–206, 2007.

[8] Z. Zhang, "Intermittently connected mobile ad hoc networks and delay tolerant networks: Overview and challenges", IEEE Survey and Tutorial, 2006, pp. 24-37.