Energy extraction from pulsed amplified stimulated emission lasers operating under conditions of strong saturation

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The output of pulsed, homogeneously broadened amplified spontaneous emission (ASE) lasers operating under conditions of strong saturation is considered. An expression for the maximum power (energy) per unit volume that may be extracted from steady-state (pulsed) systems is derived and shown to be independent of the saturation intensity. Numerical calculations of the temporal evolution of the populations in the laser levels and the growth in the ASE fluence, peak intensity, and spectral width are presented for three types of laser system: quasi steady state, short pump pulses, and self-termination. In all cases the growth in the fluence of the ASE pulse is found to agree with an analytic solution derived for conditions of strong saturation. The calculated growth in intensity of the ASE pulse is compared with expressions derived by Pert [J. Opt. Soc. Am. B **11**, 1425 (1994)] for the steady-state case. As expected, good agreement is found for the quasi-steady-state system, but in the limit of strong saturation significant differences occur for the short pump pulse and self-terminating laser systems considered. For all cases considered, the variation of the spectral width of the ASE pulse is found to be in reasonable agreement with that predicted by Pert for steady-state conditions. © 2006 Optical Society of America

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1. INTRODUCTION

The physics of mirrorless lasers operating under steady-state conditions, the output from which comprises amplified spontaneous emission (ASE), has been investigated by Allen and Peters,[1–4] Casperson,[5] and Pert.[6] That earlier work established that, for small values of the small-signal gain-length product $\alpha_0\ell$, the intensity of the ASE increased approximately exponentially with the length of the gain medium, but that for $\alpha_0\ell$ greater than approximately 15, saturation of the population inversion caused the ASE intensity to grow linearly with length. More recently, King and Pert have extended this analysis to the case of traveling-wave-pumped ASE lasers operating in the quasi-steady-state regime,[7] and Pert has also considered the transition from unidirectional to bidirectional behavior.[8]

The evolution of the spectral profile of the ASE has been shown to depend on the broadening mechanism of the gain medium. For homogeneously broadened gain media, the linewidth of the ASE decreases with $\alpha_0\ell$. In contrast, for inhomogeneously broadened gain media the linewidth of the ASE initially decreases with $\alpha_0\ell$, before rebroadening once the intensity of the ASE becomes comparable to the saturation intensity. The general case of simultaneous homogeneous and inhomogeneous broadening was considered by Pert[6] who showed that cross-relaxation effects and the strong wings of a Lorentzian homogeneous line shape could reduce or eliminate rebroadening of the ASE spectrum.

ASE lasers are particularly important in the extreme-ultraviolet (XUV) and soft-x-ray spectral regions owing to the fact that it is difficult to manufacture mirrors for such wavelengths. Furthermore, the short duration of the population inversion in such systems often means that an optical cavity would be of no advantage.

The emphasis of much of the present work on short-wavelength lasers is a reduction of the size and complexity of the visible and infrared driving lasers used to generate the population inversion of the XUV laser. This has led to the use of picosecond[9–12] and femtosecond[13–15] pump pulses, which can be delivered at relatively high pulse repetition rates from compact laser systems. A second theme is the development of techniques to guide the pump laser pulses over long lengths, thereby increasing the single-pass gain of the XUV laser. For example, recombination lasing has been demonstrated on the $n=2\rightarrow 1$ transition at 13.5 nm in Li^{2+}, driven within plasma channels up to 14 mm long formed by discharge or laser ablation of a LiF capillary.[16] Janulewicz *et al.* have investigated transient collisionally excited short-wavelength lasers driven within a plasma channel formed by a z-pinch discharge,[17] and lasing on the $5d-5p$ transition at 41.8 nm in Xe^{8+} has been demonstrated in both a 30 mm long plasma waveguide[18] and a 15 mm long multimode capillary waveguide.[19]

In this context, XUV lasers driven by optical field ionization (OFI) have been of recent interest owing to the fact that they can be driven by high-intensity femtosecond pump laser pulses. Since OFI lasers operate in the gas phase, the linewidth of the laser transition is narrow and they have relatively large optical gain cross sections. As a consequence, the saturation intensities of OFI laser transitions[15,20] can be more than 2 orders of magnitude lower than short-wavelength lasers driven in hot, dense plasmas formed by laser heating of solid or dense gas jet targets.[21,22] The low saturation intensities characteristic of OFI lasers has led some authors to conclude[20,23] that OFI lasers are not capable of generating output pulses of high energy or intensity. However, such conclusions are misleading since it has been assumed that the ASE laser pulse cannot be amplified much beyond saturation. The development of techniques to guide the pump laser radiation over long lengths means that it is now possible, at least in principle, to make lasers with very large values of $\alpha_0 \ell$. Furthermore, the use of guiding techniques to generate regions of gain with very high aspect ratios, coupled with the ability to pump the gain in a traveling-wave configuration, could allow very large values of $\alpha_0 \ell$ to be generated without lasing occurring in undesired directions, such as transverse to the axis of the gain region. For example, we have considered[24] a recombination laser operating on the $4s-3p$ transition of Ar^{7+} and have shown that, by driving this laser in a gas-filled capillary discharge waveguide, small-signal, gain-length products up to $\alpha_0 \ell \approx 1000$ might be possible, with output pulse energies of several hundred microjoules.

In this paper we consider the output of ASE lasers with very large values of the small-signal gain-length product $\alpha_0 \ell$ such that they operate under conditions of strong saturation over much of the gain region. In this limit it is possible to derive analytical expressions for the maximum power that can be extracted per unit volume of gain for both pulsed and steady-state conditions. Our approach emphasizes that, in the limit of strong saturation, the output of the ASE laser is independent of the saturation intensity, depending only on the pumping and decay rates of the laser levels.

The present paper differs from earlier work in that it considers ASE lasers operating under time-dependent conditions. Previous treatments of ASE lasers have considered steady-state or quasi-steady-state lasing only, and are therefore not applicable to several important classes of short-wavelength laser such as those operating on self-terminating transitions. A second goal of the paper, therefore, is a clarification of the conditions under which the earlier, steady-state analyses can be employed.

It is not, of course, possible to derive algebraic expressions for the intensity and spectral width of an ASE laser with arbitrary (time-dependent) pumping and general pumping geometry. We restrict our analysis, therefore, to the important case of matched traveling-wave pumping and assume that lasing is unidirectional. Expressions for the output of the laser are derived for conditions of strong saturation. For this limit, we derive an expression for the maximum rate of stimulated emission for pump pulses of an arbitrary temporal profile. This yields an upper limit on the power extraction per unit volume of gain medium.

The paper is organized as follows. In Section 2 we show that, for a steady-state system operating under conditions of strong saturation, the rate of stimulated emission is independent of the saturation intensity. An expression for the rate of stimulated emission under conditions of strong saturation is then derived for pump pulses of an arbitrary temporal profile and is shown to be independent of the saturation intensity. We then clarify the conditions on the parameters of the laser transition and the pump pulses that must be satisfied if the laser system can be considered to be operating in a quasi steady state such that a steady-state analysis may be employed. In Section 3 the radiation transfer equation for a pulsed ASE laser is solved numerically for three important classes of laser system: a quasi-steady-state system, a laser transition pumped with short pump pulses, and a self-terminating laser transition. It is shown that in each case the rate of growth of the fluence of the ASE pulse tends to the limit established in the present analysis. The results of the numerical simulations are also compared with the results of a steady-state analysis. It is found that, as expected, the steady-state analysis gives good agreement with the numerical simulations for the quasi-steady-state laser system but that significant differences occur in the other cases considered.

2. FUNDAMENTAL EQUATIONS AND ASSUMPTIONS

We consider a homogeneously broadened laser transition between an upper level 2 of energy E_2 and a lower level 1 of energy E_1. The fluorescence lifetime of level i for radiative and nonradiative transitions, including on the laser transition itself, is taken to be τ_i. The rate per atom in the upper laser level of transitions to the lower laser level by all processes other than stimulated emission is taken to be A_{21}; in the absence of collisional or nonradiative transitions, A_{21} would be equal to the Einstein A coefficient of the transition. In a similar way we define the rate per atom in the lower laser level of transitions to the upper laser level by all processes other than absorption of radiation to be A_{12}. This quantity would be zero in the absence of collisional or nonradiative transitions.

With these definitions, the rate equations for the two levels become

$$\frac{\mathrm{d}N_2}{\mathrm{d}t} = R_2(t) - W_s(t) - \frac{N_2(t)}{\tau_2} + N_1(t)A_{12}, \tag{1}$$

$$\frac{\mathrm{d}N_1}{\mathrm{d}t} = R_1(t) + W_s(t) - \frac{N_1(t)}{\tau_1} + N_2(t)A_{21}, \tag{2}$$

where $R_i(t)$ is the rate per unit volume at which atoms are pumped into level i and $W_s(t)$ is the rate of stimulated emission per unit volume.

The rate of stimulated emission per unit volume may be written in terms of the Einstein B coefficient for stimulated emission B_{21} as

$$W_s(t) = N^*(t) \int_0^\infty B_{21} g(\omega) \frac{\mathcal{I}(\omega,t)}{c} \mathrm{d}\omega = N^*(t) \int_0^\infty \sigma_{21}(\omega) \frac{\mathcal{I}(\omega,t)}{\hbar\omega} \mathrm{d}\omega, \tag{3}$$

where $g(\omega)$ is the normalized line-shape function of the laser transition; $\sigma_{21}(\omega) = (\hbar\omega/c)B_{21}g(\omega)$ is the optical gain cross section of the laser transition; $\mathcal{I}(\omega,t)$ is the intensity of radiation in the angular frequency interval ω to $\omega + \mathrm{d}\omega$; and the population inversion density is defined as $N^*(t) = N_2(t) - (g_2/g_1)N_1(t)$ in which g_i is the degeneracy of level i.

It is convenient to define a normalized line shape for the radiation $f(\omega,t)$, which may also be a function of time:

$$f(\omega,t)=\frac{\mathcal{I}(\omega,t)}{\int_0^\infty \mathcal{I}(\omega,t)\mathrm{d}\omega}=\frac{\mathcal{I}(\omega,t)}{I_T(t)}, \tag{4}$$

where $I_T(t)$ is the total intensity at time t. We may now rewrite Eq. (3) in the form

$$W_s(t)=\frac{N^*(t)}{\tau_R}\frac{I_T(t)}{I_s(t)}, \tag{5}$$

where the saturation intensity is given by

$$\frac{1}{I_s(t)}=\tau_R\int_0^\infty \frac{\sigma_{21}(\omega)f(\omega,t)}{\hbar\omega}\mathrm{d}\omega \approx \frac{\tau_R}{\hbar\omega_L}\int_0^\infty \sigma_{21}(\omega)f(\omega,t)\mathrm{d}\omega, \tag{6}$$

which is simply the usual form for the reciprocal of the saturation intensity, but averaged over the spectrum of the radiation. The approximation on the right-hand side of Eq. (6) holds if the bandwidth of the laser radiation is small compared to its mean angular frequency ω_L. Note that the recovery time τ_R has not yet been defined and that in principle any finite value may be used, provided that the same value is used in Eqs. (5) and (6). In Subsection 2.A we derive an expression for τ_R.

It will be useful for the discussions that follow to define the gain per unit length $\alpha(\omega,t)=N^*(t)\sigma_{21}(\omega)$ and the gain per unit length for the total intensity:

$$\alpha_T(t)=N^*(t)\int_0^\infty \sigma_{21}(\omega)f(\omega,t)\mathrm{d}\omega, \tag{7}$$

which is simply $\alpha(\omega,t)$ averaged over the spectrum of the laser radiation. Note that, provided that the bandwidth of the laser is small compared to the mean frequency ω_L, we may write

$$\alpha_T(t)=\frac{N^*(t)}{\tau_R}\frac{\hbar\omega_L}{I_s(t)}. \tag{8}$$

A. Results for Steady-State Pumping

Setting the left-hand sides of Eqs. (1) and (2) to zero and solving for the population inversion density, we find

$$\left[1-A_{12}\tau_1A_{21}\tau_2+\frac{I_T}{I_s}\frac{\tau_2(1-A_{12}\tau_1)+(g_2/g_1)\tau_1(1-A_{21}\tau_2)}{\tau_R}\right]N^* =R_2\tau_2\left(1-\frac{g_2}{g_1}A_{21}\tau_1\right)-\frac{g_2}{g_1}R_1\tau_1\left(1-\frac{g_1}{g_2}A_{12}\tau_2\right). \tag{9}$$

Hence, defining the relaxation time by

$$\tau_R=\frac{\tau_2(1-A_{12}\tau_1)+(g_2/g_1)\tau_1(1-A_{21}\tau_2)}{1-A_{12}\tau_1A_{21}\tau_2}, \tag{10}$$

we recover the well-known result

$$N^*_{I_T}=\frac{N^*_0}{1+I_T/I_s}, \tag{11}$$

where $N^*_{I_T}$ is the steady-state population inversion density produced by the pumping when subjected to radiation of total intensity I_T, and hence N^*_0 is the population inversion density that would be produced by the same pumping in the absence of stimulated emission on the laser transition.

From Eqs. (5) and (11) we then find that the rate of stimulated emission per unit volume is given by

$$W_s=\frac{N^*_0}{\tau_R}\frac{I_T/I_s}{1+I_T/I_s}. \tag{12}$$

1. Limit of Strong Saturation

In the limit of strong saturation, that is, $I_T \gg I_s$, we find that the population inversion density tends to zero and the rate of stimulated emission per unit volume W_s^{sat} and the power extracted per unit volume by stimulated emission $P_s^{\max}$ tend to maxima given by

$$W_s^{\mathrm{sat}}=\frac{N^*_0}{\tau_R}=\frac{R_2\tau_2\left(1-\frac{g_2}{g_1}A_{21}\tau_1\right)-\frac{g_2}{g_2}R_1\tau_1\left(1-\frac{g_1}{g_2}A_{12}\tau_2\right)}{\tau_2(1-A_{12}\tau_1)+(g_2/g_1)\tau_1(1-A_{21}\tau_2)}, \tag{13}$$

$$P_s^{\mathrm{sat}}=\hbar\omega_L W_s^{\mathrm{sat}}=\frac{\hbar\omega_L}{\sigma_{21}(\omega_L)\tau_R}N^*_0\sigma_{21}(\omega_L)=\alpha_0(\omega_L)I_s, \tag{14}$$

where in Eq. (14) we have assumed that the bandwidth of the laser radiation is small compared to its mean angular frequency. The second equality on the right-hand side of Eq. (13) emphasizes that the maximum rates of stimulated emission and energy extraction are independent of the optical gain cross section or line shape of the laser transition, as well as being independent of the spectral profile of the laser radiation. Instead, the maximum rate of stimulated emission is simply that rate that, given the pumping rates and level lifetimes, forces the population inversion density to zero. The interpretation is particularly simple for the case of an ideal four-level laser, i.e., when $\tau_1 \ll \tau_2$, since in that case $W_s^{\mathrm{sat}} \approx R_2$ and the maximum rate of stimulated emission corresponds to extracting one photon for every atom excited to the upper laser level.

2. Condition for Achieving a Steady-State Population Inversion

The requirement that W_s is positive yields the following condition for achieving a steady-state population inversion:

$$\frac{R_2}{R_1}\frac{\tau_2}{\tau_1}\frac{g_1}{g_2}\left(\frac{1-\frac{g_2}{g_1}A_{21}\tau_1}{1-\frac{g_1}{g_2}A_{12}\tau_2}\right)>1. \tag{15}$$

Given that all the parameters in inequality (15) are positive, we deduce a necessary, but not sufficient, condition for achieving a steady-state population inversion:

$$\left(\frac{1-\dfrac{g_2}{g_1}A_{21}\tau_1}{1-\dfrac{g_1}{g_2}A_{12}\tau_2}\right) > 0. \quad (16)$$

For the common case when $A_{12}=0$, this condition becomes the more familiar condition $(g_2/g_1)A_{21}\tau_1<1$.

B. Time-Dependent Solution in the Limit of Strong Saturation

We now consider the solution of the rate equations for time-dependent pumping in the limit of strong saturation. First, subtraction of the rate equations (1) and (2) gives the equation describing the evolution of the population inversion:

$$\frac{\mathrm{d}N^*}{\mathrm{d}t} = R^*(t) - \beta W_s(t) - \left[N_2(t)\left(\frac{1}{\tau_2}+\frac{g_2}{g_1}A_{21}\right) - \frac{g_2}{g_1}N_1(t)\left(\frac{1}{\tau_1}+\frac{g_1}{g_2}A_{12}\right)\right], \quad (17)$$

where the net rate of pumping of the population inversion density is given by

$$R^*(t) = R_2(t) - \frac{g_2}{g_1}R_1(t), \quad (18)$$

$$\beta = 1 + \frac{g_2}{g_1}. \quad (19)$$

In Eq. (17) it is clear that the term in square brackets represents the rate of change of population inversion density by processes other than pumping or stimulated emission.

Now, under conditions of strong saturation $N^*(t)\approx 0$, whereas $W_s(t)$ remains finite, so that

$$N_2(t) \approx (g_2/g_1)N_1(t). \quad (20)$$

Within this approximation, Eq. (17) becomes

$$\frac{\mathrm{d}N^*}{\mathrm{d}t} = R^*(t) - \beta W_s^{\mathrm{sat}}(t) - \gamma_2^* N_2(t) \approx 0, \quad (21)$$

where the rate of loss of population inversion per atom in the upper laser level is given by

$$\gamma_2^* = \frac{1}{\tau_2} + \frac{g_2}{g_1}A_{21} - \left(\frac{1}{\tau_1}+\frac{g_1}{g_2}A_{12}\right). \quad (22)$$

Note that γ_2^* can be either positive or negative.

From Eq. (21) we then find an expression for the maximum rate of stimulated emission per unit volume under conditions of strong saturation, $W_s^{\mathrm{sat}}(t)$, where

$$\beta W_s^{\mathrm{sat}}(t) = R^*(t) - \gamma_2^* N_2^{\mathrm{sat}}(t). \quad (23)$$

The rate equations for the populations of the upper and lower laser levels under conditions of strong saturation may be found by substituting approximation (20) into Eqs. (1) and (2) to yield

$$\frac{\mathrm{d}N_2^{\mathrm{sat}}}{\mathrm{d}t} = \frac{R_2(t)+R_1(t)}{1+g_1/g_2} - \bar{\gamma}_{21}N_2^{\mathrm{sat}}(t), \quad (24)$$

$$\frac{\mathrm{d}N_1^{\mathrm{sat}}}{\mathrm{d}t} = \frac{R_2(t)+R_1(t)}{1+g_2/g_1} - \bar{\gamma}_{21}N_1^{\mathrm{sat}}(t), \quad (25)$$

where

$$\bar{\gamma}_{21} = \frac{g_2(1/\tau_2 - A_{21}) + g_1(1/\tau_1 - A_{12})}{g_1+g_2} \quad (26)$$

is the mean rate of decay of the upper and lower laser levels by all transitions other than on the laser transition itself. These first-order differential equations may be solved by the use of an integrating factor to give

$$N_2^{\mathrm{sat}}(t) = \frac{\exp(-\bar{\gamma}_{21}t)}{1+g_1/g_2}\int_{-\infty}^{t}[R_2(t')+R_1(t')]\exp(\bar{\gamma}_{21}t')\mathrm{d}t', \quad (27)$$

$$N_1^{\mathrm{sat}}(t) = \frac{\exp(-\bar{\gamma}_{21}t)}{1+g_2/g_1}\int_{-\infty}^{t}[R_2(t')+R_1(t')]\exp(\bar{\gamma}_{21}t')\mathrm{d}t', \quad (28)$$

where we have assumed that the upper and lower laser levels are unpopulated at $t=-\infty$ and $R_2(-\infty)=R_1(-\infty)=0$.

From these last results, and Eq. (23), we may write a key result: an expression for $W_s^{\mathrm{sat}}(t)$ in terms of the pump rates only.

$$\beta W_s^{\mathrm{sat}}(t) = R^*(t) - \gamma_2^*\frac{\exp(-\bar{\gamma}_{21}t)}{1+g_1/g_2}\int_{-\infty}^{t}[R_2(t')+R_1(t')] \times\exp(\bar{\gamma}_{21}t')\mathrm{d}t'. \quad (29)$$

The maximum power per unit volume that may be extracted by stimulated emission is simply

$$P_s^{\mathrm{sat}}(t) = \hbar\omega_L W_s^{\mathrm{sat}}(t), \quad (30)$$

where $\hbar\omega_L$ is the photon energy of the laser transition. Hence once the pump rates of a laser system have been calculated, Eqs. (29) and (30) are sufficient to calculate the maximum power per unit volume that can be extracted by stimulated emission. The maximum energy that may be extracted per unit volume of the gain medium is then found by integrating $P_s^{\mathrm{sat}}(t)$ over the interval for which $W_s^{\mathrm{sat}}(t)$ is positive.

It is worth at this point emphasizing the assumptions made in deriving Eq. (29). The most important of these is that the level lifetimes τ_i and the rates A_{12} and A_{21} are taken to be constant in time, and therefore to be independent of the population densities of the upper and lower laser levels. It is not assumed that the pump rates are independent of the population densities of the laser levels. However, if the pump rates do depend on these populations, Eq. (29) is of limited use since one must then simultaneously solve Eqs. (27) and (28), which are transcendental. Hence, from here on we will assume that the pump rates are independent of the populations in the laser levels.

In Section 3 we relate Eq. (29) to the maximum energy that can be extracted per unit volume in a pulsed ASE laser. Before doing this, however, we examine this result in more detail.

1. Quasi Steady State

We now consider the special case in which the pumping rates change slowly relative to the decay rates of the laser levels and compare the results with those found for steady-state pumping.

We may integrate the integral in Eq. (29) by parts to give

$$\frac{R_2(t)+R_1(t)}{\beta}\frac{\exp(\bar{\gamma}_{21}t)}{\bar{\gamma}_{21}} - \int_{-\infty}^{t}\frac{\dot{R}_2(t')+\dot{R}_1(t')}{\beta}\frac{\exp(\bar{\gamma}_{21}t')}{\bar{\gamma}_{21}}\mathrm{d}t', \tag{31}$$

where the dot denotes a derivative with respect to t', and we have assumed that the pump rates are zero at $t=-\infty$. The integral in expression (31) is of the same form as that in Eq. (29), but with the replacements $R_i(t)\rightarrow\dot{R}_i(t)/\bar{\gamma}_{21}$. Hence provided that in a time $1/\bar{\gamma}_{21}$ the fractional change in the pump rates is small, we may approximate the integral in Eq. (29) by the first term of expression (31). After some algebra we then find

$$W_s^{\text{sat,slow}} = \frac{R_2(t)\tau_2[1-(g_2/g_1)A_{21}\tau_1]-(g_2/g_1)R_1(t)\tau_1[1-(g_1/g_2)A_{12}\tau_2]}{1-A_{12}\tau_1A_{21}\tau_2}\frac{1}{\tau_R}, \tag{32}$$

where the recovery time τ_R is given by Eq. (10).

This result may be interpreted by solving the rate equations (1) and (2) in the absence of stimulated emission, that is, $W_s(t)=0$. Again, assuming slowly varying pumps such that $\dot{R}_i(t)\ll R_i(t)/\tau_i$ and $\dot{N}_i(t)\ll N_i(t)/\tau_i$, it is straightforward to show that Eq. (32) may be written as

$$W_s^{\text{sat,slow}}(t) = \frac{N_0^*(t)}{\tau_R}, \tag{33}$$

where $N_0^*(t)$ is the population inversion density produced by the same pumping but with no stimulated emission. As expected, this result agrees with Eq. (13), which was obtained for the case of constant pumping rates.

Within these assumptions the power extracted by stimulated emission per unit volume of the gain medium may be written with the aid of Eq. (8) in the form

$$P_s^{\text{sat,slow}}(t) = \hbar\omega_L\frac{N_0^*(t)}{\tau_R} = \alpha_T^0(t)I_s, \tag{34}$$

where we have assumed that the bandwidth of the laser radiation is small compared to the mean angular frequency ω_L and $\alpha_T^0(t)$ is the small-signal (i.e., unsaturated) gain per unit length for the total intensity. This result is useful for calculating the maximum power that may be extracted from a laser system with slowly varying pumps in terms of the small-signal gain per unit length, which is often one of the parameters output by numerical models of a laser system. However, we stress again that $P_s^{\text{sat,slow}}(t)$ depends only on the pump rates and the decay rates of the laser levels and is independent of the optical gain cross section or linewidth of the laser transition since these factors cancel in the product $\alpha_T^0(t)I_s$.

To summarize, if the rate of pumping of level i has a characteristic duration τ_i^{pump}, the rate of stimulated emission, and hence the growth of the intensity of ASE, will be well approximated by the steady-state results provided that for both levels $\bar{\gamma}_{21}\tau_i^{\text{pump}}\gg 1$ and $\tau_i^{\text{pump}}\gg\tau_i$. In practice, A_{21} and A_{12} will usually be small compared to the fluorescence decay rates of the laser levels such that $\bar{\gamma}_{21}$ will be dominated by the shorter of the two level lifetimes. If this is the case, the first pair of conditions will automatically be satisfied if the second pair are satisfied.

We should add that if the laser system is to behave in a quasi-steady manner, the condition for achieving a steady-state population inversion [inequality (16)] must also be satisfied.

2. Self-Terminating Laser Transitions

If inequality (16) is not satisfied, the population of the lower laser level will build up throughout the pump pulse by direct pumping, radiative and nonradiative decay, and stimulated emission on the laser transition. To maintain optical gain, the rate of pumping of the upper laser level would have to increase throughout the pump pulse, which is not possible indefinitely, and hence the lasing will self-terminate during the pump pulse.

The analysis presented in Subsection 2.B gives some insight into the effect of self-termination. If the lower level decays slowly (and we may neglect A_{12}), then

$$\gamma_2^* \approx \frac{1}{\tau_2}+\frac{g_2}{g_1}A_{21} = \left(\frac{1}{\tau_2}-A_{21}\right)+\beta A_{21}, \tag{35}$$

which is positive. From Eq. (23) we then see that the maximum possible rate of stimulated emission is strongly reduced by the term $\gamma_2^*N_2(t)$, corresponding to loss of population inversion by transitions other than stimulated emission. The second equality on the right-hand side of approximation (35) has two terms: The first is the rate of decay of the upper laser level on transitions other than the laser transition; the second is the rate of loss of population inversion by transitions on the laser transition other than stimulated emission. In other words, the loss of population inversion arises straightforwardly from loss of population from the upper laser level, plus nonstimulated transitions on the laser transition itself (which both decrease the upper-level population and increase the lower-level population). For strongly self-terminating laser transitions this loss rate can be significant compared to the rate of stimulated emission, even under strong saturation, since the upper-level population can increase to large values. In general, $W_s^{\mathrm{sat}}(t)$ as calculated by Eq. (23) or Eq. (29) will eventually become negative, indicating self-termination of lasing.

3. PULSED AMPLIFIED SPONTANEOUS EMISSION LASERS

A. Analysis

In the previous sections we considered the energy extraction from the population inversion, but we have not addressed how the ASE pulse grows as it propagates along the gain medium. We consider the gain medium to be in the form of a long rod lying along the z axis, with a cross section that is sufficiently small that the amplification may be considered to occur in the z direction only. The growth of the spectral intensity of a pulse of ASE propagating from the beginning of the rod at $z=0$ to the end of the rod at $z=\ell$ is then given by

$$\frac{1}{c}\frac{\partial \mathcal{I}(\omega,z,t)}{\partial t}+\frac{\partial \mathcal{I}(\omega,z,t)}{\partial z}=N^*(z,t)\sigma_{21}(\omega)\mathcal{I}(\omega,z,t)+N_2(z,t)A_{21}\hbar\omega g(\omega)\frac{\Omega(z)}{4\pi}, \tag{36}$$

where $\Omega(z)$ is the solid angle subtended at z by the end of the rod at $z=\ell$. To find the spectral intensity as a function of time and position in the rod, Eq. (36) must be solved simultaneously with the rate equations for the laser levels 1 and 2.

The problem is simplified somewhat if we consider the gain medium to be pumped in a traveling-wave configuration. We may then define speed of light coordinates $\tau=t-z/c$ so that Eq. (36) is transformed to the simpler form

$$\left[\frac{\partial \mathcal{I}(\omega,z,\tau)}{\partial z}\right]_\tau=N^*(z,\tau)\sigma_{21}(\omega)\mathcal{I}(\omega,z,\tau)+N_2(z,\tau)A_{21}\hbar\omega g(\omega)\frac{\Omega(z)}{4\pi}, \tag{37}$$

and the rate equations have the same form as Eqs. (1) and (2), but with t replaced by τ.

Integrating over all frequencies, we find the growth equation for the total intensity of the ASE pulse:

$$\left[\frac{\partial I_T(z,t)}{\partial z}\right]_\tau=\hbar\omega_L\left[W_s(z,\tau)+N_2(z,\tau)A_{21}\frac{\Omega(z)}{4\pi}\right], \tag{38}$$

where once again we have assumed that the linewidth of the transition is small compared to its mean frequency. Equation (38) cannot be integrated directly since the terms on the right-hand side depend, through the rate equations, on the total intensity $I_T(z,\tau)$. However, in the limit of strong saturation, the right-hand side becomes a function of the pump rates only and the intensity grows according to

$$\begin{aligned}\left[\frac{\partial I_T(z,\tau)}{\partial z}\right]_\tau^{\mathrm{sat}}&=\hbar\omega_L\left[W_s^{\mathrm{sat}}(z,\tau)+N_2^{\mathrm{sat}}(z,\tau)A_{21}\frac{\Omega(z)}{4\pi}\right]\\&=\left(\frac{\hbar\omega_L}{\beta}\right)\Bigg\{R^*(z,\tau)-\left[\gamma_2^*-\beta A_{21}\frac{\Omega(z)}{4\pi}\right]\\&\quad\times\frac{\exp(-\bar{\gamma}_{21}\tau)}{1+g_1/g_2}\int_0^t[R_2(z,\tau')+R_1(z,\tau')]\\&\quad\times\exp(\bar{\gamma}_{21}\tau')\mathrm{d}\tau'\Bigg\}.\end{aligned} \tag{39}$$

Clearly if the pumping is uniform along the rod, and we may neglect the variation of $\Omega(z)$ with z, the total intensity at any point τ on the pulse grows linearly with propagation distance.

The fluence $\Gamma(z)$ of the ASE pulse is given by integrating the above over the time interval for which the right-hand side is positive. Hence in the limit of strong saturation, the fluence of the pulse grows linearly according to

$$\frac{\mathrm{d}\Gamma^{\mathrm{sat}}(z)}{\mathrm{d}z}=U_s^{\mathrm{sat}}(z), \tag{40}$$

where the maximum energy per unit volume that can be extracted from the gain medium, $U_s^{\mathrm{sat}}(z)$, is given by

$$U_s^{\mathrm{sat}}(z)=\hbar\omega_L\int\left[W_s^{\mathrm{sat}}(z,\tau)+N_2^{\mathrm{sat}}(z,\tau)A_{21}\frac{\Omega(z)}{4\pi}\right]\mathrm{d}\tau, \tag{41}$$

$$\begin{aligned}\to&=\left(\frac{\hbar\omega_L}{\beta}\right)\int\Bigg\{R^*(z,\tau)-\left[\gamma_2^*-\beta A_{21}\frac{\Omega(z)}{4\pi}\right]\\&\quad\times\frac{\exp(-\bar{\gamma}_{21}\tau)}{1+g_1/g_2}\int_0^\infty[R_2(z,\tau')+R_1(z,\tau')]\\&\quad\times\exp(\bar{\gamma}_{21}\tau')\mathrm{d}\tau'\Bigg\}\mathrm{d}\tau,\end{aligned} \tag{42}$$

and the outer integral on the right-hand side of Eq. (42) is evaluated over the interval for which the integrand is positive. In this limit the contribution from nonstimulated transitions will be small, and we may drop the second term on the right-hand side of Eq. (41):

$$\begin{aligned}U_s^{\mathrm{sat}}(z)&\approx\hbar\omega_L\int W_s^{\mathrm{sat}}(z,\tau)\mathrm{d}\tau\\&=\left(\frac{\hbar\omega_L}{\beta}\right)\int\Bigg\{R^*(z,\tau)-\gamma_2^*\frac{\exp(-\bar{\gamma}_{21}\tau)}{1+g_1/g_2}\\&\quad\times\int_0^\infty[R_2(z,\tau')+R_1(z,\tau')]\exp(\bar{\gamma}_{21}\tau')\mathrm{d}\tau'\Bigg\}\mathrm{d}\tau.\end{aligned} \tag{43}$$

B. Pulsed Amplified Spontaneous Emission Lasers

To illustrate the ideas presented above, we have compared the analytical results derived in Section 2 with numerical solutions of the rate equations (1) and (2) and the equation for the growth in the spectral intensity of the ASE pulse [Eq. (37)].

For this illustration the pump rates of the upper and lower level were taken to be Gaussian in time, i.e., of the form $R_i^{\mathrm{peak}} \exp[(-\tau/\Delta\tau_i^{\mathrm{pump}})^2]$. The degeneracies of the upper and lower laser levels were assumed to be identical, and the line shape of the laser transition was taken to have a Lorentzian profile with a full width at half-maximum $\Delta\omega_H$.

The rate equations were solved in the absence of stimulated emission to give the peak unsaturated population inversion $N_{0,\mathrm{peak}}^*$, from which we could calculate the peak unsaturated gain per unit length $\alpha_{0,\mathrm{peak}} = N_{0,\mathrm{peak}}^* \sigma_{21}(\omega_0)$, where ω_0 is the center frequency of the transition. For all cases considered, $R_2^{\mathrm{peak}}:R_1^{\mathrm{peak}}$ was taken to be 10:1. The calculated total intensity and fluence were scaled to the saturation intensity, $I_{s0} = \hbar\omega_0/\sigma_{2l}(\omega_0)\tau_R$, and saturation fluence, $\Gamma_{s,0} = \hbar\omega_0/\sigma_{21}(\omega_0)$, was calculated at the center frequency of the transition. As such, the results presented are independent of the optical gain cross section, although their detailed behavior does depend on the pumping and decay rates assumed. The length ℓ of the gain region was scaled such that $\alpha_{0,\mathrm{peak}}\ell = 1000$, and the solid angle Ω was taken to have a constant value of 10^{-6} sr.

Calculations were performed for three representative cases: quasi-steady state, short pump pulses, and self-termination. For the quasi-steady state we assumed $\tau_2:\Delta\tau_2^{\mathrm{pump}}:\Delta\tau_1^{\mathrm{pump}}:A_{21}^{-1}:\tau_1 = 1:3:3:10:0.1$. This system obeys the conditions given in Subsection 2.B.1 for achieving a steady-state population inversion and operating in the steady-state regime. For these conditions the derived parameters determining the behavior under conditions of strong saturation are $\tau_2:\tau_R:1/\bar{\gamma}_{21}:1/\gamma_2^* = 1:1.09:0.18:-0.11$.

For the short pump pulse case we took $\tau_2:\Delta\tau_2^{\mathrm{pump}}:\Delta\tau_1^{\mathrm{pump}}:A_{21}^{-1}:\tau_1, = 1:0.1:0.1:10:0.1$. This system also obeys the necessary condition for achieving a steady-state population inversion, but does not meet the condition required for quasi-steady-state operation owing to the short duration of the pump pulses. For this case the derived parameters have the same values as for the quasi-steady-state case.

Finally we examined a self-terminating system with $\tau_2:A_{21}^{-1}:\tau_1 = 1:10:50$ pumped by step-function pump pulses. The derived parameters take the values $\tau_2:\tau_R:1/\bar{\gamma}_{21}:1/\gamma_2^* = 1:46:2.2:0.93$. The long lifetime of the lower level ensures that the condition for achieving a steady-state population inversion is not met. However, the long duration of the pump pulses means that $\bar{\gamma}_{21}\tau_i^{\mathrm{pump}} \gg 1$ and $\tau_i^{\mathrm{pump}} \gg \tau_i$.

Figure 1 shows the temporal evolution of the population densities of the upper and lower laser levels at $z=0$ and $z=\ell$ and the spectrally integrated intensity of the ASE pulse at $z=\ell$ for quasi-steady state, short pump pulse, and self-termination. For the quasi-steady-state case at $z=0$, where there is no stimulated emission, the upper and lower laser levels essentially follow the pump-

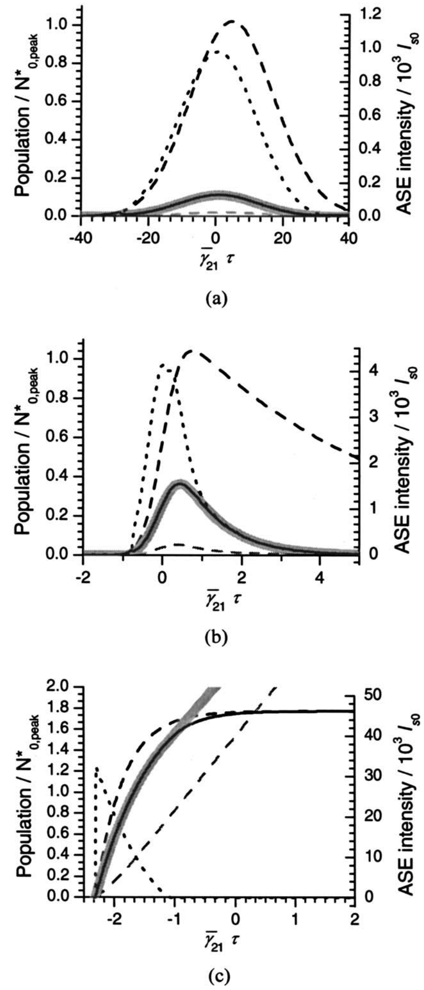

Fig. 1. Temporal evolution of the upper (black curves) and lower (gray curves) laser levels at $z=0$ (dashed curves) and $z=\ell$ (solid curves) for the cases considered in the text: (a) quasi steady state, (b) short pump pulses, (c) self-termination. In all cases the temporal profile of the total intensity of the ASE pulse at $z=\ell$ is also shown (dotted curve).

ing. However, at $z=\ell$ the intensity of the ASE pulse has grown large compared to the saturation intensity, and hence the laser transition is strongly saturated for most of the pump pulse. As such, although initially during the pump pulse the population in the upper laser level follows that calculated for $z=0$, the population in this level is quickly burnt down to a small fraction of that generated in the absence of stimulated emission. In contrast, the lower laser level increases significantly from the unsaturated value, such that for most of the pump pulse the population inversion is approximately zero; in other words, the laser transition operates in a regime of strong saturation.

The evolution of the populations in the upper and lower laser levels is quite different for the case of short pump pulses. As expected, at $z=0$ significant populations remain in the upper and lower levels after the termination of the pump pulses, and these decay by spontaneous transitions according to their fluorescence lifetimes. The population inversion generated at $z=0$ therefore persists for considerably longer than the duration of the pump pulses.

At $z=\ell$, strong saturation of the laser transition reduces the population of the upper level and increases that of the lower level, such that the population inversion is approximately zero. This situation persists beyond the termination of the pump pulses, such that the duration of the ASE pulse also extends beyond the pump pulses.

The behavior of the self-terminating system is different again. At $z=0$, the pumping initially produces a population inversion, but this becomes negative for $\bar{\gamma}_{21}\tau\approx 0.3$ owing to the buildup of population in the lower laser level. As for the other cases considered, at $z=\ell$ strong saturation causes the population inversion to be approximately zero during the ASE pulse. The strong bottlenecking of the lower level has a particularly dramatic effect on the population of the lower level, and the buildup of the population in this level causes the inversion to become negative, and hence the lasing to terminate, for $\gamma_{21}\tau\approx -1$, i.e., during the (constant) pump pulse.

In Fig. 2 the growth of the fluence of the ASE pulse is plotted as a function of propagation distance along the gain region on a log–log scale. The behavior of the three cases is broadly similar. For small values of $\alpha_{0,\text{peak}}z$, the fluence grows linearly with propagation distance, reflecting the dominance of nonamplified spontaneous emission. The growth in the fluence becomes nonlinear for $\alpha_{0,\text{peak}}z$ greater than approximately 5, and continues to grow essentially exponentially until a significant degree of saturation occurs for $\alpha_{0,\text{peak}}z$ greater than approximately 20. At larger values of $\alpha_{0,\text{peak}}z$, the growth in fluence becomes linear in $\alpha_{0,\text{peak}}z$ and tends to the limit set by strong saturation [approximation (43)]. In this limit the rate of increase in fluence with z is 6 or 7 orders of magnitude greater than the linear growth of nonamplified spontaneous emission observed for small $\alpha_{0,\text{peak}}z$. It is important to note that a significant increase in the fluence of the ASE pulse can occur after strong saturation occurs; for the cases considered, the fluence grows to be at least 3 orders of magnitude greater than the saturation fluence.

Figure 2 shows some differences between the behavior of the cases considered. For small values of $\alpha_{0,\text{peak}}z$, the growth in fluence in the self-terminating system is noticeably higher than that of the quasi-steady-state system, which in turn is greater than that observed with short pump pulses. This reflects the fact that, for a given value of the peak unsaturated gain, and hence a given population inversion, the population of the upper laser level is higher in the case of self-termination owing to the buildup of population in the lower laser level. Similarly, for short pump pulses the population in the upper laser level at the moment when the peak population inversion is reached is lower than that found in the quasi steady state. In contrast, in the limit of strong saturation the fluence is distinctly larger for the quasi-steady-state case than for the short pump pulses or self-terminating cases. This reflects the longer time for which energy can be extracted from the gain medium by saturated stimulated emission.

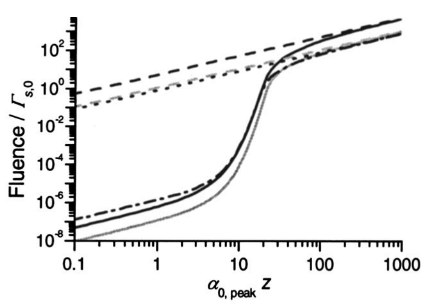

Fig. 2. Growth of the fluence of the ASE pulse as a function of distance along the gain region for the cases considered in the text: quasi steady state (black solid curve), short pump pulse (gray solid curve), and self-termination (black dashed–dotted curve). Also shown is the growth in the fluence in the limit of strong saturation [approximation (43)] for the cases of quasi steady state (black dashed curve), short pump pulse (gray dashed curve), and self-termination (black dotted curve).

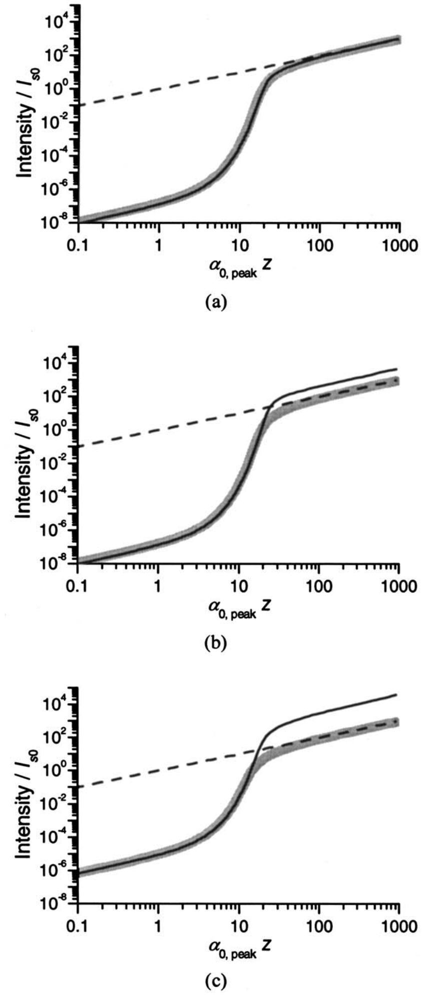

Fig. 3. Comparison of the calculated growth of the peak intensity of the ASE pulse (solid curves) as a function of distance along the gain region with the steady-state analysis of Pert (gray solid curves), and $I=\alpha_{0,\text{peak}}I_{s0}z$ (black dashed curves) for the cases considered in the text: (a) quasi steady state, (b) short pump pulse, (c) self-termination.

Figure 3 compares the calculated growth of the peak intensity of the ASE pulse with the total intensity calculated from the formulas derived by Pert[6] for a steady-state, homogeneously broadened ASE laser. In using Pert's Eqs. (31), (32), and (36) we set the unsaturated, line-center steady-state gain per unit length equal to the peak unsaturated line-center gain per unit length, $\alpha_{0,\text{peak}}$. In all cases, Pert's analysis predicts that the growth in the peak intensity becomes linear in the limit of strong saturation with a slope $\mathrm{d}I_T/\mathrm{d}z=\alpha_{0,\mathrm{peak}}I_{s0}$ as evident in Fig. 3.

As we would expect, for the quasi-steady-state case, the growth in the peak intensity follows the results of the steady-state analysis closely. For the short pump and self-terminating cases, however, the growth of the peak intensity of the ASE pulse is significantly different from that deduced by applying the steady-state analysis. For both of these cases, until the onset of saturation, that is, for $\alpha_{0,\mathrm{peak}}z$ less than approximately 10, the calculated peak intensity of the ASE pulse is in good agreement with that calculated with the steady-state analysis. However, it is seen that, once strong saturation occurs, the peak intensity grows to be significantly larger than calculated by the steady-state theory, and at a rate that is significantly larger than the limit set by $\mathrm{d}I_T/\mathrm{d}z=\alpha_{0,\mathrm{peak}}I_{s0}$. In the limit of strong saturation, the peak intensity of the ASE pulse increases at a rate that is above $\mathrm{d}I_T/\mathrm{d}z=\alpha_{0,\mathrm{peak}}I_{s0}$ by factors of 4 and 40, respectively, for the short pump pulse and self-terminating cases.

The reason for this discrepancy is, quite simply, that a steady-state theory cannot be applied to systems in which the pump pulses are short or the laser transition exhibits self-termination. Indeed, it is perhaps surprising that the agreement is so good for small values of $\alpha_{0,\mathrm{peak}}z$. The reason for the good agreement in this limit is as follows. Other than the geometric parameters of the gain medium and the lifetimes and decay rates of the laser levels, the parameters input to the steady-state theory are the population inversion, the saturation intensity, and the upper-level population density. For small values of $\alpha_{0,\mathrm{peak}}z$, the level populations are unsaturated, and hence at the peak of the ASE pulse the true rates of spontaneous and stimulated emission will be essentially the same as those found in a system operating in the steady state with the same populations in the upper and lower laser levels. As such, the peak intensity of the ASE pulse will grow according to the steady-state theory.

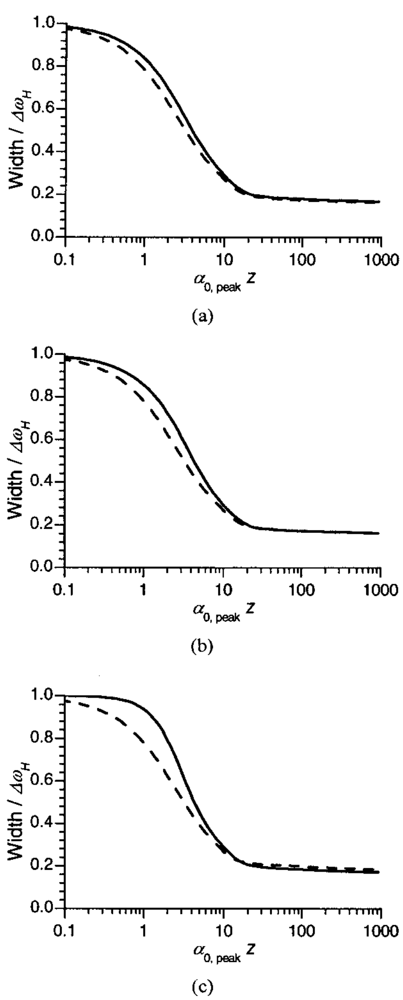

Fig. 4. Comparison of the variation of the full width at half-maximum of the fluence of the ASE pulse (solid curves) with the steady-state analysis of Pert (dashed curves) for the cases considered in the text: (a) quasi steady state, (b) short pump pulse, (c) self-termination.

The discrepancies that occur for larger values of $\alpha_{0,\mathrm{peak}}z$ should be expected. In our application of the steady-state analysis we have used the peak, unsaturated population inversion produced in a transient case (either the short pump pulse or the self-terminating case), $N^*_{0,\mathrm{peak}}$. However, this population inversion will be lower than would be achieved in a steady-state system with the same pump rates that exist at the moment the peak population inversion is achieved in the transient system: For short pump pulses, the population in the upper laser level is still increasing toward the steady-state value; for self-terminating systems, the buildup of population in the lower laser level reduces the inversion from that which would be produced in a system obeying the necessary condition for achieving a population inversion. As such, the rate of stimulated emission deduced from this population inversion by a steady-state analysis is lower than is possible in the actual system being considered. In particular, under conditions of strong saturation the rate of stimulated emission given by the steady-state analysis is $N^*_{0,\mathrm{peak}}/\tau_R$, whereas in fact the rate of stimulated emission is limited by the (higher) pump rates that exist at the peak of the ASE pulse [Eq. (23)].

Finally, in Fig. 4 we plot the variation of the full width at half maximum of the fluence spectrum and compare this with the width of the spectral intensity calculated from Pert's Eq. (30). As expected, the ASE pulse exhibits spectral narrowing with propagation through the gain medium. The spectral width calculated by the steady-state theory is narrower than the width of the ASE pulse for all z, reflecting the fact that for much of the duration of the ASE pulse the gain is lower than the peak value.

4. DISCUSSION

The analysis above has made several simplifying assumptions that deserve comment. First, and most fundamentally, we have used a rate equation approach and hence have ignored coherent phenomena such as Rabi oscillations and power broadening. The effects of coherent phenomena on the properties of steady-state ASE lasers have been considered by Griem and Moreno.[25] These effects will be unimportant provided that the rate of stimulated emission is small compared with the linewidth of the laser transition, i.e., if $\sigma_{21}(\omega_0)I_T/\hbar\omega_L \ll \Delta\omega_H^{-1}$, where $\Delta\omega_H$ is the homogeneous linewidth of the transition, a condition that may be written as $I_T \ll (\Delta\omega_H \tau_R) I_s$. For most situations of interest, τ_R, which is characteristic of the level lifetimes, is much larger than $\Delta\omega_H^{-1}$ and consequently rate equations are expected to be valid even for total intensities much greater than the saturation intensity. However, it is acknowledged that in principle the onset of coherent phenomena limits the validity of the results presented here, and so should be borne in mind when analyzing a real system.

We have also ignored absorption of the ASE pulse as it propagates through the gain medium. Of particular importance are sources of absorption that either do not saturate or that have saturation intensities that are large compared to that of the laser transition of interest. In these circumstances the ASE pulse may be amplified only to intensities such that the saturated gain per unit length is reduced so as to equal the absorption per unit length, since the net gain of the pulse will then be zero. However, including saturable absorption into our analysis is unlikely to be enlightening since it would require the rate equations for the absorber to be incorporated; this would complicate the results considerably and introduce several additional parameters that in any real situation are unlikely to be known. Furthermore, in many situations of interest the optical cross section of the absorption transition will be orders of magnitude smaller than that of the laser transition. In this case the saturation intensity of the absorption will be large compared to that of the laser transition; the unsaturated absorption per unit length will be small compared with the unsaturated gain per unit length; and hence the intensity of the ASE pulse can be amplified to many times the laser saturation intensity. In any case, approximation (43) is useful in giving the maximum possible energy that can be extracted per unit volume of gain.

5. CONCLUSIONS

In summary, we have analyzed the output of pulsed, homogeneously broadened ASE lasers pumped in the traveling-wave configuration and developed an expression for the maximum possible energy per unit volume that can be extracted by stimulated emission. Conditions were derived on the pumping and decay rates of the laser levels that must be satisfied for the system to operate in the quasi-steady state such that they may be treated by earlier steady-state analyses.

Numerical solutions for the temporal evolution of the populations in the laser levels and the growth in the ASE fluence, peak intensity, and spectral width were presented for three types of laser system: quasi steady, state, short pump pulses, and self-termination. In all cases the growth in the fluence of the ASE pulse was found to agree with the upper limit derived in the present paper. For the short pump and self-terminating systems considered, the calculated growth in intensity of the ASE pulse was found to differ significantly from that calculated by a steady-state analysis.

Our approach emphasizes that the energy of the ASE pulse is not limited by the saturation intensity of the laser transitions, but is instead determined by the pump and decay rates of the laser levels and the single-pass, small-signal gain-length product of the laser medium. As such it would be useful to incorporate into computer codes that calculate the gain of such systems an evaluation of the maximum energy per unit volume that may be extracted from the gain medium under conditions of strong saturation. If the assumptions made in deriving approximation (43)—namely, that the level lifetimes are constant and the pump rates are independent of the population densities in the laser levels—are valid, these results may be used to calculate the maximum extractable energy per unit volume. In cases where the underlying assumptions are not met, the maximum extractable energy may be calculated using existing kinetic codes by artificially introducing additional transitions on the laser transitions that maintain the population inversion at zero, as we have discussed[24] recently in an analysis of a recombination laser transition in Ar^{7+}.

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