Subcarrier Allocation for Multicast Services in Multicarrier Wireless Systems with QoS Guarantees

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Abstract— **The throughput of conventional multicast transmission in wireless systems is limited by the user with the worst channel quality in the multicast service group. The subcarrier allocation for multicast services in multicarrier systems is a feasible solution to overcome the capacity limitation by exploiting the frequency diversity among subcarriers. However, most of the current subcarrier allocation algorithms are limited to unicast services. In this paper, we propose an optimal subcarrier allocation algorithm for multicast services with Quality of Services (QoS) guarantees. A low-complexity suboptimal algorithm is also proposed, which includes three steps: Conservative Allocation, Greedy Step and Iterative Enhancement. Simulation results show that the proposed algorithms significantly outperform the conventional multicast transmission scheme while at the same time guaranteeing the minimum data rates of all users. Moreover, simulation results also show that the performance difference between the optimal and suboptimal algorithms is small.**

I. INTRODUCTION

Supporting multicast/broadcast services becomes a trend of the next-generation mobile cellular networks. MBMS (Multimedia Broadcast and Multicast Service) [1] introduced by 3GPP in Release 6 and MBS (Multicast and Broadcast Service) [2] defined in IEEE802.16e are the representative mechanisms to support multicast/broadcast services, which adopt multicast transmissions to efficiently utilize radio resources. However, the main problem of multicast transmissions is that the spectrum efficiency is driven by the worst-case scenario. Therefore, although some users in a multicast service group have better channel qualities than others, they must be subject to the same transmission rate decided by the user with the worst channel quality. This leads to the capacity limitation of conventional multicast transmission schemes [3].

To overcome this capacity limitation, one solution is to exploit the hierarchical multicast transmissions based on some scalable video coding schemes [4] [5]. These types of coding schemes can encode the raw video data to a base layer containing essential information for the decodability of the video and enhancement layers including more detailed information for enhancing the quality of the video. In the hierarchical transmission mechanisms, the number of layers received by a user is dependent on the user's channel conditions. More layers can be received by the users experiencing better channel conditions to produce better video quality. Therefore, the throughput of conventional multicast schemes can be improved by using the hierarchical transmission mechanisms.

Recently, some research has been done on the hierarchical techniques for multicast transmissions, which can be classified into physical layer mechanisms and MAC layer mechanisms. For example, multi-resolution coding is a physical layer mechanism which can increase throughput of multicast transmissions by using non-uniform constellations [6] [7]. For MAC layer mechanisms, the subcarrier allocation for multicast services in multicarrier systems (eg. OFDMA systems) is the feasible solution to achieve higher throughput by exploiting the frequency diversity among subcarriers. Although the problem of subcarrier allocation for unicast transmission has been studied extensively [8] [9] [10], there are many differences between the subcarrier allocation algorithms for unicast and multicast services. The essential difference is illustrated in Fig.1. In Fig.1(a), the users receive unicast services, so one subcarrier can be allocated to only one user. If the users receive multicast services and belong to one multicast group, the subcarrier will be shared by more than one user, as shown in Fig.1(b). Consequently, the data rate assigned to a subcarrier should be decided by a group of users which share the subcarrier, not by one user as in the algorithms for unicast services.

However, the research on subcarrier allocation algorithms for multicast services is limited. The subcarrier/bit allocation algorithms were developed in [3] for multicast services to increase the throughput. In [11] dynamic subcarrier and power allocation algorithms for multicast groups were proposed. However, the above subcarrier allocation algorithms for multicast services do not take the Quality of Service (QoS) requirements into considerations. Therefore, these algorithms can not guarantee the minimum data rate required by the base layer for all the users in a multicast service group. As a result, the video can not be decoded by the users without receiving the base layer correctly since it is impractical to assume that any combination of the layers can be decoded.

In this paper, we focus on the Subcarrier Allocation method for Multicast services (SA-M) to improve multicast throughput while at the same time satisfying the QoS requirements of all users. The optimal algorithm for SA-M is derived based on Integer Programming. However, the computational complexity of the optimal algorithm increases exponentially with the number of subcarriers and users. Therefore, a low-complexity suboptimal algorithm is proposed which includes three steps: 1) implement the initial allocation to satisfy the

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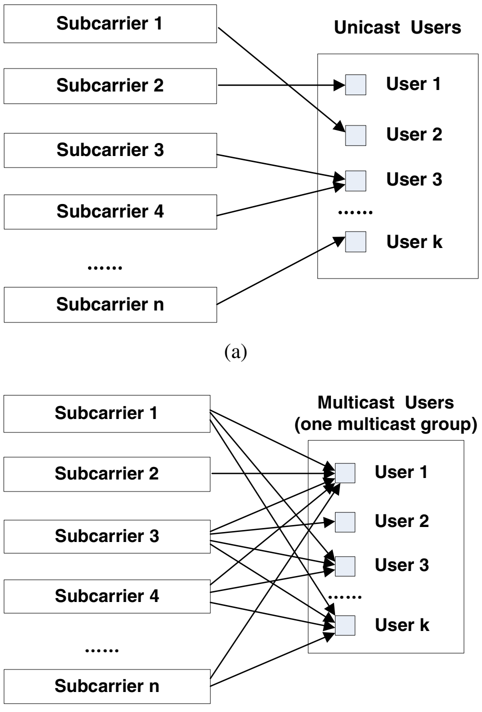

(a)

(b)

Fig. 1. Subcarrier Allocation for unicast and multicast services

QoS requirements of all the users; 2) assign the unallocated subcarriers by a greedy method and 3) improve the total data rate of the previous allocation by iterative subcarrier swapping. Compared with the conventional multicast transmission mechanism, the proposed optimal and suboptimal algorithms for SA-M can achieve better performance for multicast throughput without violating the QoS requirements of all the users that receive the multicast services.

The rest of the paper is organized as follows. In Section II, the system model is introduced and the SA-M problem is formulated. In Section III, we analyze the SA-M problem and propose the optimal and suboptimal algorithms. Simulation results are given and discussed in Section IV. Section V concludes the paper.

II. SYSTEM MODEL

In this paper, we consider a downlink multicarrier wireless system with a single base station (BS) and multiple users. The frequency band is divided into N subcarriers, which are shared by K users in a multicast service group receiving the same services. During a subcarrier allocation cycle, the scheduler in BS assigns each subcarrier to more than one user in the multicast service group, so the data rate of each subcarrier should be the minimum among the achievable data rates of all the users that share the subcarrier.

The original multicast service is encoded into hierarchical data with a base layer and some enhancement layers. The base layer is necessary for decoding and the quality of the multicast service increases proportionally as more of the enhancement layers are received. Therefore, the QoS requirement of each user is to obtain the required bit rate of the base layer, denoted as R_{base}. For each allocation cycle, the algorithm should guarantee that every user can receive the minimum data rate which is calculated based on R_{base}. The calculation method can be found in Section III.C.

Next, we list some notations used in this paper and then formulate the subcarrier allocation problem for multicast services.

k	Index of users in the multicast service group, $k \in (1, 2, \ldots, K)$.
n	Index of subcarriers for the downlink channel, $n \in (1, 2, \ldots, N)$.
$c_{k,n}$	The achievable data rate of user k at subcarrier n in the current time slot for a given BER and transmission power.
c_n	The data rate that is assigned to subcarrier n for the multicast service group.
$x_{k,n}$	Indication of subcarrier n allocation to user k. $x_{k,n}$ equals to 1 if subcarrier n is allocated to user k, otherwise, $x_{k,n}$ is equal to 0.
r_k^{min}	Minimum data rate that user k should receive in the allocation cycle.

The total bandwidth of the wireless channel is B Hz and if we use f Hz to denote the bandwidth of each subcarrier, we obtain $f = B/N$. The achievable data rate of user k at subcarrier n, i.e., $c_{k,n}$, can be calculated as follows [12]:

$$c_{k,n} = f \log_2(1 - \frac{1.5}{ln(5 * BER)} \rho_{k,n}) \tag{1}$$

where $\rho_{k,n}$ is the signal to noise ratio (SNR) of user k on subcarrier n. Assuming that the power is equally distributed over all the subcarriers, the maximum feasible data rate is only decided by the current channel quality represented by SNR and BER. The users measure the current Channel State Information (CSI) based on the received signals and then send CSI back to BS via a feedback channel. It is assumed that the scheduler in BS knows exactly the CSI at the beginning of each subcarrier allocation cycle, that is, $c_{k,n}$ is available at the beginning of each allocation cycle.

The goal of the subcarrier allocation problem for multicast services is to maximize the multicast throughput while guaranteeing the QoS requirements of all the users that receive the multicast services. Based on the definitions above, the problem can be formulated as follows:

$$\max_{c_n, x_{k,n}} \sum_{k=1}^{K} \sum_{n=1}^{N} c_n x_{k,n} \tag{2a}$$

$$s.t. \quad \sum_{n=1}^{N} c_n x_{k,n} \geq r_k^{min}; \quad k = (1, 2, \ldots, K) \tag{2b}$$

$$\frac{1}{c_n} = \max_k \frac{x_{k,n}}{c_{k,n}}; \qquad n = (1, 2, \ldots, N) \tag{2c}$$

$$1 \leq \sum_{k=1}^{K} x_{k,n} \leq K; \quad n = (1, 2, \ldots, N) \tag{2d}$$

Objective function (2a) is the total multicast data rate as a result of the current subcarrier allocation cycle. Constraints (2b) are the minimum data rates obtained by users in the current allocation cycle to ensure that the users can decode the multicast data correctly. Constraints (2c) ensure that the assigned data rate to subcarrier n is the minimum of the data rates that the selected users can achieve at this subcarrier. Each subcarrier can be allocated to more than one user as depicted in Fig.1(b), which is the interpretation of Constraints (2d). Clearly, Problem (2) is a 0-1 integer programming (IP) problem and it is nonlinear because of the max function. We will describe the optimal solution and a practical suboptimal solution of this problem in the next section.

III. SUBCARRIER ALLOCATION FOR MULTICAST SERVICES

In this section, we first discuss the optimal algorithm for Problem (2). Subsequently, we propose a suboptimal solution to alleviate the burden of high computational complexity of the optimal algorithm.

A. Optimal Algorithm

In order to obtain a typical IP problem, Constraints (2c) should be converted into linear ones. We replace the max function in Constraints (2c) by searching over all the possible choices of $c_{k,n}$ to linearize the Constraints (2c) as follows:

$$c_n \cdot \frac{x_{1,n}}{c_{1,n}} \le 1, c_n \cdot \frac{x_{2,n}}{c_{2,n}} \le 1, \dots, c_n \cdot \frac{x_{k,n}}{c_{k,n}} \le 1; \quad n = (1,2,\dots,N) \tag{2c$'$}$$

Moreover, we need to replace c_n and $x_{k,n}$ $(k = 1,2,\dots,K; n = 1,2,\dots,N)$ by a new indicator as the decision variables. Since the supported data rates of each subcarrier in a practical system are usually the limited number of constant values, denoted as d_m $(m = 1,2,\dots,M)$, the new indicator can be defined as follows:

$$\gamma_{k,n,m} = \begin{cases} x_{k,n}, & c_n = d_m \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

With the new decision variables $\gamma_{k,n,m}$, $c_n x_{k,n}$ can be replaced by $\sum_{m=1}^{M} d_m \gamma_{k,n,m}$. In addition, Constraints (2d) should be redefined as the constraints of $\gamma_{k,n,m}$:

$$\begin{aligned} &\{1 \le \sum_{k=1}^{K} \gamma_{k,n,1} \le K \ and \ \sum_{m \ne 1} \sum_{k=1}^{K} \gamma_{k,n,m} = 0\} \ or \\ &\{1 \le \sum_{k=1}^{K} \gamma_{k,n,2} \le K \ and \ \sum_{m \ne 2} \sum_{k=1}^{K} \gamma_{k,n,m} = 0\} \ or \\ &\cdots\cdots \\ &\{1 \le \sum_{k=1}^{K} \gamma_{k,n,M} \le K \ and \ \sum_{m \ne M} \sum_{k=1}^{K} \gamma_{k,n,m} = 0\}; \\ &\qquad\qquad n = (1,2,\dots,N) \end{aligned} \tag{2d$'$}$$

Based on the above analysis and definitions, Problem (2) can be converted into a linear IP problem as follows. This problem can be solved using software tools such as CPLEX.

$$\begin{aligned} &\max_{\gamma_{k,n,m}} \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{m=1}^{M} d_m \gamma_{k,n,m} \\ &s.t. \sum_{n=1}^{N} \sum_{m=1}^{M} d_m \gamma_{k,n,m} \ge r_k^{min}; \quad k = (1,2,\dots,K) \\ &\quad \sum_{m=1}^{M} d_m \cdot \frac{\gamma_{1,n,m}}{c_{1,n}} \ge 1, \sum_{m=1}^{M} d_m \cdot \frac{\gamma_{2,n,m}}{c_{2,n}} \ge 1, \\ &\quad \dots, \sum_{m=1}^{M} d_m \cdot \frac{\gamma_{k,n,m}}{c_{k,n}} \ge 1; \quad n = (1,2,\dots,N) \\ &\quad and \ Constraints (2d') \end{aligned}$$

The above IP problem is NP-hard and its complexity increases exponentially with the number of constraints and variables. Therefore, the optimal algorithm can only work offline and is not suitable for the real-time operation in practical systems, especially when a fast fading channel is considered. Thus, a suboptimal algorithm with low complexity is proposed to solve the problem.

B. Suboptimal Algorithm

The suboptimal algorithm is composed of three steps: Conservative Allocation, Greedy Step and Iterative Enhancement. Firstly, the conservative allocation step generates an initial version of the allocation which can satisfy the QoS requirements of all the users in a fast way. Next, the unallocated subcarriers are assigned to maximize their aggregate data rate in a greedy step. As a last step, the iterative enhancement based on subcarrier swapping improves the total data rate of the previous allocation. Although the suboptimal algorithm can not achieve the best result, its complexity is significantly reduced compared to that of the optimal algorithm.

B.1 Conservative Allocation

The main idea of the conservative allocation is to assign subcarriers to the users that must be served, i.e. $r_k^{min} > 0$, in the current cycle to guarantee their minimum data rates. Assume that the number of these users is K_1 $(K_1 \le K)$. In order to get the allocation sequence of these K_1 users, the allocation urgency degree of user k, denoted as u_k, is defined as:

$$u_k = \frac{r_k^{min}}{\sum\limits_{n=1}^{N} c_{k,n}} \tag{4}$$

It can be seen that the urgency degree of a user is in the direct ratio to its minimum data rate required and in the inverse ratio to its channel condition. The larger the value of u_k is, the more urgently user k needs to be served. In the conservative allocation, we allocate as few subcarriers as possible for each user to satisfy its QoS requirement in the descending order of the urgency degree. That is, the user with the most urgent allocation requirement is allocated first. The conservative allocation step is summarized as Algorithm 1.

Algorithm 1 Conservative Allocation

```
1: Let $f_n$ be the indication whether subcarrier $n$ is allocated. $f_n$ equals to 0 if subcarrier $n$ has been allocated and 1 if not. Set $r_k$ as the current data rate of user $k$. $r_k = 0(k = 1, 2, \ldots, K_1)$ and $f_n = 1(n = 1, 2, \ldots, N)$ at the beginning.
2: Calculate $u_k$ according to (4) and sort them in the descending order, assuming the order is $u_1 \geq u_2 \geq \ldots \geq u_{K_1}$ without loss of generality.
3: From user $k$ = 1 to $K_1$, repeat the following steps:
    3.1: Update $r_k^{min}$ for users $k$ as follows: for all the subcarriers $n \in (1, 2, \ldots, N)$, if $f_n$ = 0 and $c_n \leq c_{k,n}$, $r_k^{min} = r_k^{min} - c_n$.
    3.2: Rank the achievable data rates of user $k$ at all the subcarriers in the descending order and get the set $A = \{c_{k,l_1}, c_{k,l_2}, \ldots, c_{k,l_N}\}$ where $l_1, l_2, \ldots, l_N \in (1, 2, \ldots, N)$. Let $N' = min\{n' : \sum_{n'=1}^{N'} c_{k,l_{n'}} f_l \geq r_k^{min}\}$. For $n \in (l_1, l_2, \ldots, l_{N'})$, if $f_n$ = 1, then set $c_n = c_{k,n}$, $f_n$ = 0.
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B.2 Greedy Step

The greedy step is proposed to allocate the remaining subcarriers that have not been allocated after the first step. The allocation principle in this step is to maximize the total data rate of every remaining subcarrier. Assume that the remaining subcarriers construct the set $B = \{n_1, n_2, \ldots\}$ and $i \in B$. We define

$$\phi_i(d_m) = \sum_{k=1}^{K} d_m \delta_{k,i} \tag{5}$$

as the aggregate rate of all the users at subcarrier i when the data rate of subcarrier i is $d_m(m = 1, 2, \ldots, M)$, where $\delta_{k,i}$ is 1 if $c_{k,i} \geq d_m$ and 0 otherwise. Based on (5), we can find the data rate assigned to subcarrier i at which the aggregate rate of all the users is maximized as follows:

$$c_i = \arg\max_m \phi_i(d_m) \tag{6}$$

Once c_i is found, the users which can share subcarrier i are decided.

B.3 Iterative Enhancement

In order to improve the performance of the previous allocation which may produce the total data rate lower than the optimal algorithm, we can perform the iterative subcarrier swapping [13] in the enhancement step. In each iteration, the data rate of every subcarrier can be adjusted to increase the total data rate. Subcarrier allocation is modified according to the adjustment that leads to the maximum throughput increase without violating the QoS requirements of all the users. The algorithm of iterative enhancement is described as Algorithm 2.

Algorithm 2 Iterative Enhancement

```
1: Calculate the current data rate $r_k$ of all the users $(k = 1, 2, \ldots, K)$ after the previous two steps;
2: For each subcarrier $n$ $(n = 1, 2, \ldots, N)$, perform the following:
    2.1: For each $d_m (d_m \neq c_n)$, set $c_n$ to $d_m$, update $r_k$ of all the users. If $r_k \geq r_k^{min}(k = 1, 2, \ldots, K)$, calculate $\Delta\phi_n(d_m) = \phi_n(d_m) - \phi_n(c_n)$ according to (5);
    2.2: Find $\Omega_n = max\Delta\phi_n(d_m)$, $m \in \{1, 2, \ldots, M\}$;
3: Select $\Omega = max\Omega_n$, $n \in \{1, 2, \ldots, N\}$;
4: If $\Omega > 0$, modify the assigned data rate of the corresponding subcarrier and update $r_k$ of all the users, then go to Step 2; otherwise the algorithm stops.
```

C. Data Rate Tracking

Both optimal and suboptimal algorithms rely on the availability of r_k^{min} at the beginning of each allocation cycle. r_k^{min} is calculated based on the history information of the subcarrier allocation and the data rate requirement of every user in terms of R_{base}. Next we provide a method to calculate r_k^{min} based on the sliding window mechanism. [10] We use T to denote the size of the sliding window and we have:

$$\frac{T-1}{T} r_k^{-} + \frac{1}{T} r_k^{min} = R_{base} \tag{7}$$

where r_k^{-} represents the data rate that user k has received before the current allocation cycle. We can obtain r_k^{min} based on (7):

$$r_k^{min} = T * R_{base} - (T-1) r_k^{-} \tag{8}$$

IV. SIMULATION RESULTS

In this section, we study performance of the proposed optimal and suboptimal algorithms by simulation. We assume that there are 32 subcarriers (subchannels) in the wireless channel. However, we implement the optimal algorithm using 8 subcarriers because its complexity is too high with the large number of subcarriers and users. The channel is a frequency selective Rayleigh fading channel. The set of supported data rates of one symbol is assumed as {38.4kbps, 76.8kbps, 153.6kbps, 307.2kbps, 614.4kbps} in the simulation when the subcarrier frequency spacing is about 32kHz and the symbol duration is 100us. The required bit rate of the base layer is 256kbps [5].

A. Multicast Data Rate

Fig. 2 shows the total multicast data rate as a function of the number of users when the number of subcarriers $N = 8$ and the number of supported data rates $M = 3$. We compare the proposed optimal and suboptimal algorithms with the Conventional Multicast Transmission (CMT) scheme. In the CMT scheme, each subcarrier is shared by all the users of a service group and the data rate assigned to the subcarrier is the minimum among the achievable data rates of all the users. As shown in Fig.2, the optimal and suboptimal algorithms significantly outperform the CMT scheme in the total data rate. This is because the optimal and suboptimal algorithms can make the users with better channel conditions obtain higher data rates while in the CMT scheme, the data rates of all the users in a service group are decided by the user

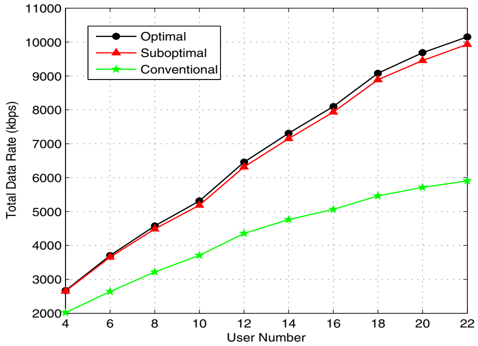

Fig. 2. Total Data Rate vs User Number (N=8,M=3)

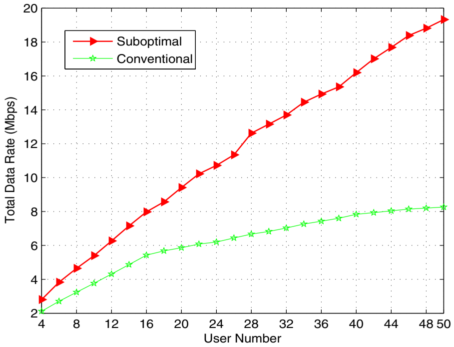

Fig. 3. Total Data Rate vs User Number (N=32,M=5)

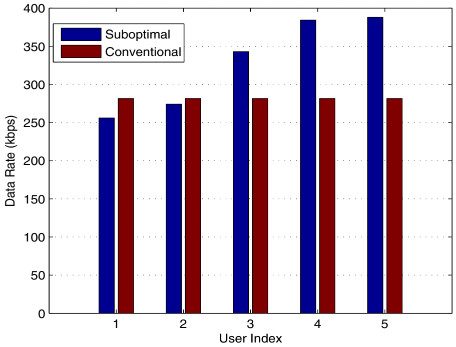

Fig. 4. User Data Rate vs Channel Quality

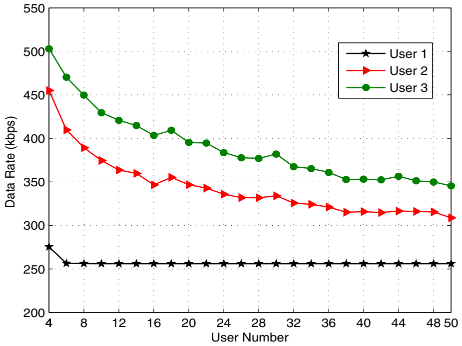

Fig. 5. User Data Rate vs User Number

with the worst channel quality. It also can be seen from Fig.2 that the performance difference between the optimal and suboptimal algorithms is within about 3%, which indicates that the suboptimal algorithm performs well.

Fig. 3 depicts the comparison of the suboptimal algorithm and the CMT scheme in the total data rate when the number of subcarrier $N = 32$ and the number of supported data rates $M = 5$. The optimal algorithm is not included in this comparison due to its high complexity. We can see that the suboptimal algorithm always outperforms CMT scheme. Moreover, the performance gap becomes larger as the number of users increases, which means that the suboptimal algorithm will be more useful when the number of users is larger.

B. Minimum Data Rate Guarantee

Fig.4 shows the average data rates received by users with different channel qualities. There are 20 users in the multicast service group. The channel quality becomes better as the user index increases from user 1 to user 5. The other 15 users' channel qualities are uniformly distributed between that of user 1 and user 5. We can see that the suboptimal algorithm can assign the higher data rates for the users with better channel qualities such as user 3-5 while guaranteeing the minimum data rates of user 1 with the worst channel quality. However, the data rate assigned to each subcarrier in the CMT scheme is based on the minimum data rate of all the users, so the data rates of all the users are the same no matter how good or bad their channel qualities are. This is also the reason why the algorithms proposed in the paper can achieve higher data rates as shown in Fig.2 and Fig.3.

In Fig.5, user 1, user 2 and user 3 have the increasing channel qualities. Although the data rates of the three users decrease when the number of users increases, their minimum average data rates are always above the required bit rate of the base layer, i.e., 256kbps. The results show that the proposed suboptimal algorithm can guarantee the users' minimum data rate requirements.

V. CONCLUSION

Although there are some subcarrier allocation algorithms for multicast services in multicarrier systems, few of them take the QoS requirements of users into considerations. Therefore, we propose new subcarriers allocation methods including both optimal and suboptimal algorithms to improve the multicast throughput while at the same time guaranteeing the QoS requirements. Simulation results show that the proposed optimal and suboptimal algorithms achieve better performance in comparison to the conventional multicast transmission schemes without violating the QoS requirements of all the users. Moreover, it is shown that the suboptimal algorithm is an efficient solution in practice.

REFERENCES

[1] Introduction to Multimedia Broadcast and Multicast Services(MBMS)in Radio Access Network, 3GPP TS 25.346, Rev. v7.4.0, Jul.2007.

[2] IEEE Standard for Local and Metropolitan Area Networks Part 16: Air Interface for Fixed and Mobile Broadband Wireless Access Systems, 2005. IEEE Standard 802.16 Working Group.

[3] Changho Suh and Jeonghoon Mo, *"Resource Allocation for Multicast Services in Multicarrier Wireless Communication," IEEE Transactions on Wireless Conmmunications*, vol.7, issue.1, pp.27-31, Jan.2008.

[4] Coding of Audio-Visual Objects-Part2: Visual, ISO/IEC Std.14496-2, May 2004.

[5] F. Wu, S. Li, and Y. Q. Zhang, *"A framework for efficient progressive fine granularity scalable video coding," IEEE Transactions on Circuits and Systems for Video Technology*, vol.11, no.3, pp.332-344, Mar.2001.

[6] A.Correia et al., *"Multi-resolution Broadcast/Multicast Systems for MBMS", IEEE Transactions on Brodcasting*, vol.53, pp.224-234, Mar.2007.

[7] M.B.Pursley and J.M.Shea, *"Multimedia multicast wireless communications with phase-shift-key modulation and convolutional coding," IEEE J. Select. Areas Comm.*, vol.17, pp.1999-2010, Nov. 1999.

[8] Wong, I.C., Zukang Shen, Evans, B.L., Andrews, J.G., *"A low complexity algorithm for proportional resource allocation in OFDMA systems,"* in *Proc. of SIPS 2004*, pp 1-6.

[9] I. Kim, H. L. Lee, B. Kim, and Y. H. Lee, *"On the use of linear programming for dynamic subchannel and bit allocation in multiuser OFDM,"* in *Proc. of IEEE GLOBECOM '01*, vol.6, 2001, pp.3648-3652.

[10] G. Fang, Y. Sun, J. Zhou, J. Shi, Z. Li, *"Subcarrier Allocation for OFDMA Wireless Channels Using Lagrangian Relaxation Methods"*, in *Proc. of IEEE GLOBECOM*, Oct.2006.

[11] J.Liu, W. Chen, Z. Cao, Letaief K.B., *"Dynamic Power and Sub-Carrier Allocation for OFDMA-Based Wireless Multicast Systems"*, in *Proc. of IEEE ICC*, May.2008.

[12] S. T. Chung, A. J. Goldsmith, *"Degrees of freedom in adaptive modulation: a unified view", IEEE Trans. Commun.*, vol.49, no.9, Sep.2001.

[13] Mustafa Ergen, Sinem Coleri, and Pravin Varaiya, *"QoS Aware Adaptive Resource Allocation Techniques for Fair Scheduling in OFDMA Based Broadband Wireless Access Systems", IEEE Transactions on Brodcasting*, vol.49, no.4, Dec.2003.