Globally Controlled Quantum Wires for Perfect Qubit Transport, Mirroring, and Computing

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We describe a new design for a q wire with perfect transmission using a uniformly coupled Ising spin chain subject to global pulses. In addition to allowing for the perfect transport of single qubits, the design also yields the perfect "mirroring" of multiply encoded qubits within the wire. We further utilize this global-pulse generated perfect mirror operation as a "clock cycle" to perform universal quantum computation on these multiply encoded qubits where the interior of the q wire serves as the quantum memory while the q-wire ends perform one- and two-qubit gates.

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The development of protocols for transmitting quantum states is a particularly important problem in quantum computation. The ability to produce q wires would allow quantum information to be moved around within a quantum processor. In the initial work [1,2], the transport of quantum states through *unmodulated* spin chains was examined and less-than-perfect transport fidelities were found [1,3–7]. This is due to the dispersion of the quantum information along the chain [8]. Much work has since ensued searching for perfect q-wire transport schemes and briefly we can categorize these into: (1) if the nearest-neighbor couplings between systems comprising the q wire are set to very specific values [6,7], one can achieve perfect transport. (2) One can achieve near perfect transport by encoding the quantum information into low-dispersion wave packets, or by encoding or decoding via conditional quantum logic across multiple q wires [3,8–10]. (3) Use "gapped systems," where the q-wire ends are only weakly coupled to a strongly intercoupled interior of the q wire [11], to achieve near perfect transport. (4) Other possibilities include teleportation of the quantum information along the q wire by measurements [12], encoding into solitonlike excitations [13], or use quantum cellular automata concepts [14]. Besides the transport of single qubits, of more interest is the capability of the q wire to transport entire qubit registers via "quantum mirror wires" [15]. Here an unknown multiqubit quantum state, when encoded at one end of the wire is transmitted to the other end, but in reverse order, $\rho^{i_1 i_2\cdots i_N}_{j_1 j_2\cdots j_N} \in \mathcal{H}^1\otimes\mathcal{H}^2\otimes\cdots\mathcal{H}^N \rightarrow \tilde{\rho} = \rho^{i_N i_{N-1}\cdots i_1}_{j_N\cdots j_1}$. Experimental proposals for q wires include Josephson junction arrays [16], molecular magnet wires [17], quantum nano-electromechanical systems [18], and tunnel-coupled electronic quantum dots [19].

As well as demonstrating that globally addressed q wires can yield perfect qubit transport and perfect multiqubit mirroring we will also show that they can be used to execute universal quantum computation. We achieve this via a combination of the application of selective local unitaries on the ends of the q wire and homogenous local unitaries [HLUs [20]] (or global pulses) on the entire q wire. The use of HLUs alone to perform quantum computation has been examined by a number of authors [2,21–23]. In all but the last of these, the application of HLUs alone is not sufficient to implement universal quantum computation and some structuring of the q wire is typically required, e.g., two or three types of cells in the q wire. Our hybrid approach using HLUs and end-system selective addressing has a number of benefits over pure HLU computing. We require no structuring of the q wire while the use of robust composite pulses [24] can greatly reduce the effects of any static variations in the intersystem coupling strengths. Finally, to our knowledge, no fault tolerant quantum error correction scheme has been found for pure HLU quantum computation. It is our hope that such a scheme might be more feasible in our hybrid design. It may be that such q wires could comprise both the computational and communication resources within a quantum processor and possibly lead to greater simplifications in the required technology.

State transfer and Ising interactions.—The simplest approach to state transfer in a q wire is to simply swap qubits in neighboring locations, repeating the process on alternating pairs of qubits until the desired state has reached the end of the q wire, at which point no further swaps are performed. Building on this idea we follow the steps outlined in Fig. 1 to arrive at the circuit (f), which transports unknown state $|q_1\rangle$ using simultaneously applied Hadamard operations $\bar{H} \equiv \prod_{j=1}^{N} H^j$, and controlled phase operations $\overline{CZ} \equiv \prod_{j=1}^{N-1} CZ^{j,j+1}$. Complete transport through an N-system q wire with the initial state $|q_1 + 0 + 0\cdots\rangle$ requires the application of $\overline{CZ}\cdot(\bar{H}\cdot\overline{CZ})^{N-1}$ global operations. From (f) it would appear that such transport will require the very particular initial state $|q_1 q_2 q_3\cdots q_N\rangle = |q_1 + 0\cdots\rangle$, where q_a, $a = 2, \ldots, N$ are the $|+(0)\rangle$, pure states alternately. However, this is not the case as we show below and any initial state of these other systems will suffice (even completely mixed states).

The execution of (f) in Fig. 1 requires the application of the *global* pulses $\bar{H}$ and $\overline{CZ}$. We have assumed that the interior q-wire systems are identical and $\bar{H}$ is generated via

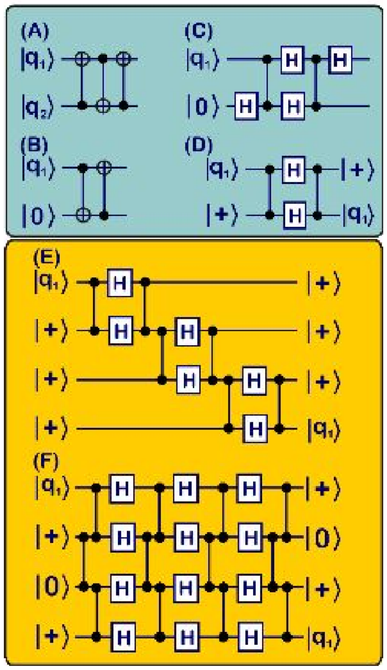

FIG. 1 (color online). We derive the network for globally controlled, perfect state transfer via several relatively simple steps. (a) A SWAP-gate, (b) same but with only $|q_1\rangle$ to SWAP. Using $\text{CNOT}^{a,b} = H^b CZ^{a,b} H^b \phi$, where CZ is the $\pi/2$ phase gate, we arrive at (c), and recoding the known input state $|0\rangle \rightarrow |+\rangle$, we have (d). (e) Chaining these operations together to perfectly transport an unknown state. (f) The input qubits $|q_1 q_2 \cdots q_n\rangle = |++\cdots+\rangle$ are obtained via the input state $|+0\cdots+\rangle$, and homogenous applications of Hadamard and CZ.

the single-qubit operations on the degenerate interior systems with selective application of Hadamards simultaneously on the end systems (Note: typically interior and end systems will possess different resonant frequencies due to their difference in neighbor interactions.) To execute $\overline{CZ}$, we assume a uniform Ising interaction between all q-wire systems. Similar to the Hadamard global operation, we find that the execution of $\overline{CZ}$ consists of natural evolution under the Ising Hamiltonian together with single-qubit operations which are uniform across the q wire except at the end systems. Allowing the uniform interaction $H_{\text{Ising}} = J\sum_{a=1}^{n-1} \sigma_z^a \sigma_z^{a+1}$, to run for $t = \frac{\pi\hbar}{4J}$, we obtain $U_{\text{Ising}} = \exp(-i\frac{\pi}{4}\sum \sigma_z^{(a)}\sigma_z^{(a+1)})$. We can expand an individual $CZ^{a,b} = \frac{1}{2}(I + \sigma_z^a + \sigma_z^b - \sigma_z^a\sigma_z^b)$, and using this we can expand the full $\overline{CZ} = \prod_{a=1}^{N-1} CZ^{a,a+1} = \prod_{a=1}^{N-1}\frac{1}{2}(I + \sigma_z^a + \sigma_z^{a+1} - \sigma_z^a\sigma_z^{a+1})$. The generating Hamiltonian for this transformation is $H = \hbar g \sum_{a=1}^{n-1} \frac{1-\sigma_z^{(a)}}{2}\frac{1-\sigma_z^{(a+1)}}{2}$. This can be expanded and using U_{Ising}, we see that

$$\overline{CZ} = \exp\left[-i\frac{\pi}{4}(\sigma_z^{(1)} + \sigma_z^{(N)})\right]\left[\prod \exp\left(i\frac{\pi}{2}\sigma_z^{(a)}\right)\right]U_{\text{Ising}}$$
$$= R_z^{(1)}\left(\frac{\pi}{4}\right)R_z^{(N)}\left(\frac{\pi}{4}\right)\left[\prod R_z^{(a)}\left(-\frac{\pi}{2}\right)\right]U_{\text{Ising}}, \quad (1)$$

where $R_z^{(a)}(\theta)$ is a z rotation, performed on the system at position a. The single-qubit operations in (1) consist of a $-\pi/2$ homogenous $z$rotation on each q-wire system except for the end systems which have additional $\pi/4$ z rotations (via selective pulses). Since these z rotations commute with U_{Ising}, we can choose to execute them either before or after the Ising interaction. A magnetic field along the z axis could be used to perform the rotations while the Ising interaction is running; however, this may not be optimal and we can choose to wrap these z rotations in with the following $\bar{H}$ global operation (except for the last application of the $\overline{CZ}$, where these rotations must either be executed or passed on to the next computational element following the q-wire transmission). The evolution in (f) consists of repetitions of $\bar{H}\cdot\overline{CZ}$. Setting $\overline{CZ} = \prod_{a=1}^{N} e^{i\theta_a\sigma_z^{(a)}} U_{\text{Ising}}$, where $\theta_a = \pi/4$, $a = 1, N$, or $\theta_a = \pi/2$, $a \neq 1, N$ and noting that $H\sigma_z H = \sigma_x$, and $HH = \mathbb{I}$, we have

$$\bar{H}\cdot\overline{CZ} = \left(\prod_{a=1}^{N} e^{i\theta_a\sigma_x^a}\right)\bar{H}U_{\text{Ising}}.$$

We now use $-iH = \exp(-i\pi\sigma_x)\exp(-i\pi/2\sigma_y)$ to obtain

$$\bar{H}\cdot\overline{CZ} = \left[(-i)^N \prod_{a=1}^{N} e^{i\chi_a\sigma_x^{(a)}} e^{-i\pi/2\sigma_y^{(a)}}\right]U_{\text{Ising}},$$

where $\chi_a = -3\pi/4$, $a = 1$, N and $\chi_a = -\pi/2$, $a \neq 1$, N. Thus the combination of a homogenous application of CZ gates on neighboring sites on the wire, together with an ensuing application of local Hadamard gates on all sites, can be generated via the standard Ising interaction followed by "bang-bang" type homogenous local operations on each qubit (apart from two σ_x operations applied at the end sites—which we assume can be selectively addressed apart from the bulk of the wire). Thus the perfect transport circuit of Fig. 1(f) requires only global addressing of the q wire, an Ising interaction which is uniform along the q wire and selective manipulation of the q-wire end systems.

Perfect quantum mirrors.—The above perfect transport seems to depend having the particular initial state $|q_1 + 0\cdots\rangle$. Although this state allowed us to easily derive the transport circuit in Fig. 1(f), it is not necessary as the total operation $\mathcal{S} \equiv (\bar{H}\cdot\overline{CZ})^{N+1}$ constitutes a perfect quantum mirror which reverses the spatial location of any quantum information encoded on the q wire. To see this we take, with no loss in generality, the initial state of an N-system q wire to be in a pure product state with $|\psi\rangle_{\text{init}} \sim \cdots \otimes |q_k\rangle \otimes$, where any pure state of the kth system can be expressed as $|q_k\rangle = \alpha_k|0\rangle_k + \beta_k|1\rangle_k = (\alpha_k + \beta_k\sigma_x^{(k)})|0\rangle_k$. To prove mirror transport it suffices to prove mirror transport of the initial state operations, i.e., $\mathcal{S}\sigma_x^{(k)} = \sigma_x^{(N-k+1)}\mathcal{S}$ (and similarly for $\sigma_z^{(a)}$). To prove these identities we make use of the following rules for propagating these operators through the global operations $\overline{CZ}$ and $\bar{H}$:

$$\overline{CZ}\sigma_z^{(a)} = \sigma_z^{(a)}\overline{CZ}, \qquad \overline{CZ}\sigma_x^{(1)} = \sigma_x^{(1)}\sigma_z^{(2)}\overline{CZ},$$

$$\overline{CZ}\sigma_x^{(N)} = \sigma_z^{(N-1)}\sigma_x^{(N)}\overline{CZ},$$
$$\overline{CZ}\sigma_x^{(a)} = \sigma_z^{(a-1)}\sigma_x^{(a)}\sigma_z^{(a+1)}\overline{CZ}, \qquad \bar{H}\sigma_z^{(a)} = \sigma_x^{(a)}\bar{H}. \tag{2}$$

Using these rules one can follow the propagation of $\sigma_x^{(a)}$ (or $\sigma_z^{(a)}$), through the global operations, e.g., $(\bar{H}\cdot\overline{CZ})^2\sigma_x^{(5)} = \sigma_x^{(3)}\sigma_z^{(4)}\sigma_x^{(5)}\sigma_z^{(6)}\sigma_x^{(7)}(\bar{H}\cdot\overline{CZ})^2$. The propagation can be more easily understood through a graphical representation [see Fig. 2(a)]. Using these rules and the graphical representation one can show $\mathcal{S}\sigma_x^{(a)} = \sigma_x^{(N-a+1)}\mathcal{S}$, $\mathcal{S}\sigma_z^{(a)} = \sigma_z^{(N-a+1)}\mathcal{S}$. We see from Fig. 2(a) that the propagation typically undergoes a period of expansion until the pattern hits the nearest q-wire end. It then continues to expand in the other direction while remaining "stuck" at the end it has impacted. Following two applications of $\bar{H}\cdot\overline{CZ}$ after impact the pattern reflects off this nearest wire end and then the process of impact, sticking, and reflection repeats off the other end of the q wire. Following $N+1$ applications of clocking operation, $\bar{H}\cdot\overline{CZ}$, the initial product state of the q wire undergoes a perfect spatial inversion about the wire's midpoint and consequently the inversion of any initial state of the q wire occurs after a full *cycle* of $\mathcal{S} = (\bar{H}\cdot\overline{CZ})^{N+1}$ operations. The construction of perfect quantum mirror transport using only global operations may need only modest technological developments to become possible in the near future in a variety of physical implementations.

Single-qubit gates.—Besides quantum transport we show how a q wire can perform universal quantum computation. We make full use of the capability to separately manipulate the ends of the q wire. We refer back to Fig. 2(a) and we note that the pattern resulting from a single-qubit operation acting on the initial state impacts a horizontal edge of this pattern in a series of four cells. To arrange that these edge qubit patterns do not overlap we now assume an initially padded qubit register, i.e., $|\psi\rangle_{\rm init} = |q_1 + q_2 + q_3 + \cdots\rangle$. To execute universal quantum logic we demonstrate single- and two-qubit gates. To achieve the former, the execution of a general qubit rotation, $U^{(a)}(\alpha,\beta,\gamma) \equiv R_z(\alpha)R_y(\beta)R_z(\gamma)$, on any qubit q_a, we use three full mirror cycles of $\mathcal{S}$. During the first cycle, to execute $R_z(\gamma)$, on q_a, we apply this single-qubit operation on an edge at the impact points $L_i^{(a)}$, in Fig. 2(b). Following one round of $\mathcal{S}$, which leaves the qubit register spatially reversed along the q wire, we apply the global operation $\bar{H}_y \equiv \prod_{a=1}^N H_y$, where $H_y \equiv R_x^{(a)}(\pi/2)$. We then apply a second round of $\mathcal{S}$, and during this apply $R_z(\beta)$ (at $L_i^{(a)}$). Following this second round of $\mathcal{S}$ we globally apply $\bar{H}_y$ and using $H_yR_z^{(a)}(\theta)H_y = R_y^{(a)}(\theta)$, we see that $R_y(\beta)$ is executed on q_a. In the third round of $\mathcal{S}$, we again apply $R_z(\alpha)$ (at $L_i^{(a)}$) to arrive at $U^{(a)}(\alpha,\beta,\gamma)$. Following from the fact that adjacent qubit patterns do not overlap when

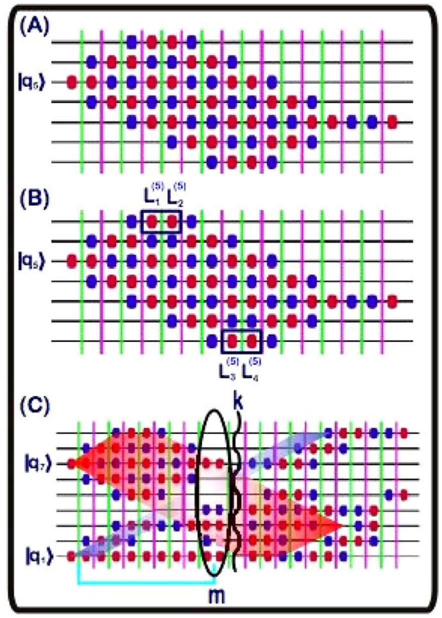

FIG. 2 (color online). Method of executing single- and two-qubit gates. (a) Mirror transport of the local unitary operations X (red), and Z (blue), acting on $|q_5\rangle$. The global pulses are $\overline{CZ}$ (green vertical bars), and $\bar{H}$ (purple vertical bars). (b) To execute single-qubit gates on $|q_4\rangle$ we apply σ_z operations on the ends at the times $L_1^{(5)},\ldots,L_4^{(5)}$. (c) Two-qubit gate between control $|q^c\rangle = |q_1\rangle$, and target $|q^t\rangle = |q_7\rangle$. Colored overlays blue (red) are the transport patterns of an initial X on $|q_1\rangle$ ($|q_7\rangle$). Underlying pattern is the simultaneous transport of both where the $|q_1\rangle$ trap is on during the indicated period (shown in light blue). The $|q_1\rangle$ pattern moves along the bottom edge of the graph (c) until it reaches time step m, where the trap is turned off. At m the target pattern begins to impact the trapped pattern and when this occurs we globally apply $\overline{CZ}\cdot\overline{CZ}$ but lift off the trap at the end site for a short time to yield a $CZ[\theta] = {\rm diag}\,(1,1,1,e^{i\theta})$, or a controlled phase gate between the two patterns. At time k we can either reverse the temporal order of the global operations or continue forward, retrapping the control pattern to execute a number of controlled phase gates targeting any qubit pattern which impacts that trapped pattern.

they impact an end (due to our use of buffer states), we can pipeline the above single-qubit operation and are able to execute $\prod_{a=1}^N U^{(a)}(\alpha_a,\beta_a,\gamma_a)$, i.e., arbitrary single-qubit operations on all qubits encoded within the q wire, using three rounds of $\mathcal{S}$, using edge operations and global $\bar{H}_y$.

Two-qubit gates.—To execute two-qubit gates we utilize the end-system control to apply decoupling pulses selectively to either end system but more simply one can apply a $R_x(\pi/2)$ pulse to the end spin midway through the Ising gate to average out the Ising interaction completely, or use selective pulses to move an end-system qubit to an "off-line" storage memory using techniques such as those recently demonstrated in a nitrogen-vacancy-^{13}C coupled system [25]. By decoupling off an end system we artificially shorten the q wire and by continuing to apply the global operations $\bar{H}\cdot\overline{CZ}$, (while omitting the Hadamard on the decoupled end site), we can cycle the remaining qubits within this shortened q wire. Since we are unable to apply the local $R_z^{(N-1)}(-\pi/4)$ necessary to complete $\overline{CZ}$ on the shortened q wire, an unwanted $R_z^{(N-1)}(\pi/4)$ is introduced each time we $\overline{CZ}$.

To execute a control-phase gate on qubit $|q^t\rangle$, controlled by the state of $|q^c\rangle$, both encoded in different spatial sites within the q wire, we wait until the X pattern from $|q^c\rangle$ impacts an end of the q wire whereupon we apply a decoupling pulse sequence to trap this X pattern at the end of the q wire. The target qubit pattern will cycle forward and will reach a configuration where it commences to impact the trapped X pattern from $|q^c\rangle$ [see Fig. 2(c)]. Then, instead of $\overline{CZ} \cdot \bar{H} \cdot \overline{CZ}$, we apply the Ising interaction for a time period $\tau_{2\pi}$, to yield $\overline{CZ} \cdot \overline{CZ}$ on all systems within the q wire, while lifting the decoupling of the end system for a time $\tau_\theta = \tau_{2\pi} * (\theta/2\pi)$, during this global operation. The result of this is to execute the identity operation on all qubits within the q-wire bar, the end-system qubit, and its immediate neighbor which suffer a control-θ operation and along with a $R_z^{(N-1)}[(\pi - \theta)/4]$. All the operations up to this point, except for the control θ and its $R_z^{(N-1)}[(\pi - \theta)/4]$, will later be reversed, and so we can tolerate the additional rotations as long as they commute with the control-θ operator between the last two qubits.

As rotations introduced at the edge of the chain propagate away from the edge, only the $R_z^{(N-1)}(-\pi/4)$ introduced with the final $\overline{CZ}$ prior to the controlled-θ gate does not commute with it. To overcome this, we apply a global $R_z(\pi/4)$, cancelling the rotation on qubit $N - 1$, but introducing rotations on qubits $1...(N - 2)$. Clearly, these commute with the controlled-θ gate between the last two qubits.

We now wish to undo everything except the controlled-θ gate, to return to the initial state where now $|\tilde{q}^c\rangle \otimes |\tilde{q}^t\rangle = R_z^{(t)}[(\pi - \theta)/4]CZ[\theta]|q^c\rangle \otimes |q^t\rangle$, and where $CZ[\theta] = \text{diag}(1, 1, 1, e^{i\theta})$. This can be done by applying, in reverse order, the inverse of each gate used to reach this point. $\bar{H}$ and $\overline{CZ}$ are, obviously, their own inverses, and $R_z(-\pi/4)$ is the inverse of the global $R_z(\pi/4)$. It is important to note, however, that $\overline{CZ_D}$, the result of applying $\overline{CZ}$ with the end-system qubit decoupled, is not its own inverse. This requires us to use $\overline{CZ_D'} = R_z^{(1)}(-\frac{\pi}{4})[\prod R_z^{(a)}(\frac{\pi}{2})]U_{\text{Ising}}^3$ instead of $\overline{CZ_D}$ when reversing the trapping sequence. This will remove any unwanted rotations introduced by not correcting the extra $R_z^{(N-1)}(\pi/4)$ caused at each $\overline{CZ_D}$.

However, more usefully, instead of reversing the temporal order of the global pulses we reverse only as far as the global $R_z(\pi/4)$ before continuing forward in the cycling evolution of the q-wire patterns, while still keeping the end system trapped to repeat the execution of a further $CZ[\theta_2]$, on another target qubit. Continuing in this fashion we can execute $CZ[\theta_1, \theta_2, \cdots, \theta_{n-1}]|q^c, q_1^t, q_2^t, \cdots q_{n-1}^t\rangle$, for a q wire encoding n qubits. At the end of a full cycle we release the trap and return the control qubit back to its original spatial location in the q wire. This multitarget 2-qubit gate can provide significant savings when it comes to executing more complicated quantum circuits such as the quantum fast Fourier transform.

Quantum fast Fourier transform.—The quantum Fourier can be written as $\text{QFT} = H_N W^{N-1} H_{N-1} \ldots W^2 H_2 W^1 H_1$, where $W^x = |0_x\rangle\langle 0_x| \otimes I + |1_x\rangle\langle 1_x| \otimes \prod_{j\neq x} W_j$ with $W_j = R_j(\pi/2^{j-x})$ for $j > x$ and $W_j = I_j$ otherwise. Clearly, W^x is composed of $N - x$ individual controlled phase (CPHASE) gates. Thus the quantum Fourier transform can be constructed using $(N - 2)(N - 1)/2$ controlled phase gates and N Hadamard gates. As has been shown earlier, arbitrary CPHASEs ($CZ[\theta]$) can be easily implemented in this scheme. Furthermore, all the CPHASE gates controlled by a particular qubit can be performed in at most a single mirroring cycle of the system. Thus, each W_x term takes only a single mirror cycle. The corresponding Hadamard gate can also be performed during this cycle, reducing the time required to perform a QFT to only N-1 mirror cycles of the system.

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