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The stability of the mesospheric plasma layer

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The presence of micron and sub-micron size dust in the Earth's summer mesopause are a possible cause of electron density depletion. Whereas electrons in this weakly ionized and weakly magnetized layer are frozen in the magnetic field, the ions and dust are highly diffusive. This relative drift between the plasma particles will cause a current in the medium. The presence of such a current can destabilize the plasma layer with a growth rate of the order of Alfvén frequency. Since required current density for the onset of this instability is on the order of $J \gtrsim 0.03\,\mathrm{A/m^2}$, it is quite unlikely that such a strong current is present in the mesosphere. However, owing to the prevailing ambiguity of measurements, the existence of such a current is not completely ruled out. © *2011 American Institute of Physics*. [doi:10.1063/1.3671958]

I. INTRODUCTION

The Earth's polar mesosphere is the site of many interesting phenomena which are poorly understood. For example, in the cold summer mesopause (~80–90 km), the water ice particles are visible from the ground as noctilucent clouds (NLC).[1] Typical density of these ice particles, which we shall call dust, can vary between ~10 to $10^3\,\mathrm{cm^{-3}}$ and their radius can vary between ~10–100 nm.[1,2] The measurements during polar mesospheric summer echoes (PMSE) conditions showed a pronounced depletion of the electron number density. In fact, the electron density can decrease by an order of magnitude where strong PMSE are observed. The depletion of electron density, usually called electron *bite–outs*, is a generic feature of the polar summer mesopause.[3,4] Although very first observations of electron bite-outs led to the suggestion that dust particles could act as a sink for electrons,[3] it was not until the first direct measurement of the dust charge[5,6] that this idea was widely accepted. Clearly, the electron bite-outs are directly correlated to the presence of charged dust at PMSE altitudes.

Although presence of dust in the mesosphere could be due to several reasons including volcanos and human activities, ablation and re–condensation of meteoric smoke particles (MSP) is the most important nucleation source for the formation of ice particles.[7] Models of dust production,[8] which considered the height profile and size distribution of the meteoric smoke particles predicts a dense $\gtrsim 10^3\mathrm{cm^{-3}}$, nanometre size dust concentration between 80–90 km. Note that unlike NLC, where owing to the thermal structure of the polar mesosphere, the ice particles/dust causes electron bite-outs in the narrow layer, the role of MSP in metal oxide containing low nanometre size particle formation and ensuing electron bite-outs is much more general. In fact, electron bite-outs is quite generic outside the NLC sessions as well in the D-region.[9]

The dust particles in the upper and lower mesosphere are either neutral or charged. However, large ($\gtrsim$20nm) ice crystals are often negatively charged in the NLCs. The presence of large dust significantly augments the electron recombination rate which is believed to cause the electron biteouts. In fact by capturing electrons from the background plasma, dust particles leave their *finger prints* in the weakly ionised and weakly magnetised background plasma.[1] As a result, ohm diffusivity of the ambient plasma will increase.

The diffusive charging model is widely regarded as the viable mechanism of the dust charging in the polar mesosphere.[10] Notwithstanding the fact that many dust particles can acquire positive charge,[11] due perhaps to photoemission, the average charge on the dust is negative owing to large mobility of electrons. The microscopic grain charging model suggests that the charge on the dust linearly increases with the radius. For example, typical charge Z on the dust will be $-1e$ for particles with radius $a \in [1, 10]$ nm, $-2e$ for 30 nm particles, and $-3e$ for 50 nm particles.[1] Here, e is the electron charge. Note that the nanometre size dust is much heavier than the plasma particles (dust mass $\sim 10^{-17}\,\mathrm{g} - 10^{-16}\,\mathrm{g}$) and such a mass disparity will cause the relative drift between the lighter plasma particles and dust. After forming at higher altitude (~90 km), the largest of dust particles will settle to lower altitude where they are visible as NLC whereas smaller dust is *visible* only through strong radar echoes.[1] Therefore, the presence of charged dust can cause a vertical current that will be aligned to the ambient magnetic field in the layer.

Although physical argument for the existence of a current/electric field in the dusty PMSE layer appears quite compelling, the measurement of such an electric field has proved notoriously difficult. For example, initial claim of ~1 V/m electric field in PMSE layer was not supported by subsequent laboratory experiments.[12] Some observations suggest that a *real geophysical* field $\lesssim 10\mathrm{mV/m}$ do exist in this layer.[13] Large, downward field ~3 V/m have also been reported in NLC environment without PMSE.[14] Current density $\sim 5 \times 10^{-5}$ A/m^2 in the sporadic E region (~90–125 km) have been reported in the past.[15] However, dynamics of this region is controlled mainly by windshear and quasiperiodic oscillations.[16]

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The Hall current, also called the equatorial electrojet, is typically ~600 km wide in the north-south direction across magnetic equator where an enhanced current flows in the horizontal, east-west direction at an altitude of ~100 km in the E region of the ionosphere. The observed daily magnetic field variation in the narrow belt of the geomagnetic equator is attributed mainly to the dynamo currents at 90–120 km region. The explanation of these currents is based on the Hall effect. The profile of the plasma conductivity in the equatorial electrojet region can be severely altered by the presence of charged dust since the presence of dust significantly affects the electron abundance.

The cause of the ionospheric irregularities is often attributed to the current driven, electrostatic instability[17,18] and it is believed that the electrojet is unstable due to the relative streaming of the plasma particles across the magnetic field. This Hall current instability (Farley-Buneman instability) has been observed around the equatorial zone or within the polar cap.[16] Note that Farley-Buneman instability treats the magnetic field as passive dynamical quantity, notwithstanding the observed daily variation of the magnetic fields in the narrow belt of the geomagnetic equator at 90–120 km. In the present work, we shall investigate the current driven electromagnetic instability where the ambient current is aligned to the background magnetic field and is caused by the relative drift between plasma particles.

The 80–120 km region of Earth's ionosphere is weakly ionised and weakly magnetised with neutral number density ($\gtrsim n_n = 10^{14}\mathrm{cm}^{-3}$) at 80 km far exceeding the ion number density ($n_i = 10^3\,\mathrm{cm}^{-3}$). The density of ~60 nm charged aerosol particles in a NLC layer at 83 km is $n_d \sim 2\times 10^2\,\mathrm{cm}^{-3}$.[19] The sub–visible small dust number density could be similar to the ion number density. In a weakly ionized medium, often plasma inertia is neglected and a linear relationship between the electric field $\mathbf{E}$ and plasma current $\mathbf{J}$, $\mathbf{E} = \sigma \times \mathbf{J}$ is derived, where σ is the conductivity tensor. The neglect of plasma inertia implies that the plasma dynamical response frequency is much smaller than respective gyrofrequencies. Thus, only low frequency wave can be considered in such a framework. The effect of dust on various conductivities can be easily worked out.[20] The weakly ionized, collisional plasma dynamics can be investigated either in the multi-fluid or the single fluid framework. Whereas, multi-fluid framework is well suited to describe the high frequency fluctuations,[21] the multi-fluid set of equation can be reduced to single fluid, MHD like equations to investigate the low frequency behaviour of the medium. The collision not only leads to the damping of the high frequency waves but in the low frequency limit it can also *gel* the medium together. Clearly, when the dynamical response time is much smaller than the collisional time, the multicomponent set of equations can be reduced to a single fluid description.[22]

II. BASIC MODEL

We model the 80–90 km layer of the Earth as a multicomponent, weakly ionized and weakly magnetized plasma consisting of plasma particles, charged and neutral dust, and neutral particles. Though both positive and negative ions are present in the mesosphere,[23] we shall consider only positive ions. Further, positive ions are of two types[11]: (1) simple ions, NO^+, N_2^+, and O_2^+ and (2) cluster ions ($H^+(H_2O)_n$, $n \le 10$). As has been noted above, the presence of dust will cause increased recombination rate which varies for simple and cluster ions between[24] 10^{-7}–$10^{-5}\,\mathrm{cm}^{-3}\ \mathrm{s}^{-1}$. The photoionisation of the dust by solar radiation may not be important in the polar summer mesosphere[1,2] and thus we shall neglect it.

The weakly ionized plasma layer is highly collisional and in order to quantify relative importance of various collision frequencies, we shall give their values in the sporadic E-layer. The electron-neutral, electron-ion, and ion-neutral collision frequencies are[16]

$$\nu_{en} = 5.4\times 10^4\, n_{n+13}\, T_{+2}^{1/2}, \quad \nu_{ei} = 5\, n_{e+3}\, T_{+2}^{-3/2}, \quad \nu_{in} = 5.8\times 10^3\, n_{n+13}\, A_{20}^{-1/2}. \tag{1}$$

Here, A is the mean neutral molecular mass in atomic mass units and $A_{20} = A/20$, $n_{n+13} = n_n/10^{13}\,\mathrm{cm}^{-3}$, $n_{e+3} = n_e/10^3\,\mathrm{cm}^{-3}$, and $T_{+2} = T/10^2\,K$. Since $\nu_{ei} \ll \nu_{en}$, we have neglected the electron-ion momentum exchange term in Eq. (8).

The dust-neutral collision rate is[25]

$$< \sigma v >_{dn} = 5\times 10^{-7} T_{+2}^{\frac{1}{2}} a_{-6}^2 \quad \mathrm{cm}^3\mathrm{s}^{-1}, \tag{2}$$

with $a_6 = a/10^{-6}\,\mathrm{cm}$, the dust-neutral collision frequency becomes

$$\nu_{dn} = 40\, n_{n+13}\, \mathrm{s}^{-1}. \tag{3}$$

Here, we have assumed $m_n = 20\, m_p$ where m_p is proton mass and $m_d = 4\times 10^{-18}$ g for $\rho_d = 1\,\mathrm{gm/cm}^{-3}$. The inelastic ion capture rate by the negatively charged grain and the inelastic electron sticking to the neutral grain is given as[26,27]

$$\alpha_{id} = \pi a^2 \left(\frac{8\, k_B T}{\pi m_i}\right)^{1/2} \left[1 + \left(\frac{e^2}{a\, k_B T}\right)\right] P_i S_i, \quad \alpha_{ed0} = \pi a^2 \left(\frac{8\, k_B T}{\pi m_e}\right)^{1/2} P_e S_e. \tag{4}$$

Here, we have assumed $T_e = T_i = T$ and

$$P_i \equiv 1 + \left[\frac{2}{(a\, k_B T/e^2) + 2}\right]^{1/2}, \quad P_e = 1 + \left(\frac{\pi e^2}{2\, a\, k_B T}\right)^{1/2}, \tag{5}$$

are Draine-Sutin factor due to electrostatic polarization of dust by electric field of the approaching plasma particles.[28] For $a = 10\,\mathrm{nm}$, $T = 100\,K$, the contribution of the Drain-Sutin factor to the ion-dust collision rate is ~2. For electron-neutral dust collision, this factor is ~6. Assuming that the sticking probabilities $S_e = S_i = 1$, the inelastic dust-ion $\nu_{di}^* = n_i\, \alpha_{id}$ and neutral dust–electron $\nu_{ed0}^* = n_e\, \alpha_{ed0}$ collision frequencies becomes

$$\nu_{di}^* = 3.4\times 10^{-3}\, n_{i+3}\, P_{i_2} a_{-6}\, T_{+2}\, \mathrm{s}^{-1}, \quad \nu_{d0e}^* = 1.2\times 10^{-2} n_{e+2}\, P_{e_6} a_{-6}\, T_{+2}\, \mathrm{s}^{-1}, \tag{6}$$

where $P_{i_2} = P_i/2$ and $P_{e_6} = P_e/6$ and we have assumed $m_i = m_n$. A comparison of the inelastic, Eq. (6) and elastic, Eq. (3) collision frequencies suggest that for micron-sized dust, dust-neutral collision dominates inelastic dust-plasma collision. A comparison between Eqs. (3) and (6) shows that inelastic collision, which are responsible for the dust charge fluctuation is very small in comparison with the dust-neutral particle collision.

The dynamics of such a weakly ionised, highly collisional plasma can be described in the framework of following multifluid equations. The continuity equation is

$$\frac{\partial \rho_j}{\partial t} + \nabla \cdot (\rho_j \mathbf{v}_j) = 0. \tag{7}$$

Here, ρ_j is the mass density and $\mathbf{v}_j$ is the velocity of the various plasma components and neutrals. The source and sink terms on the right hand side has been neglected which implies that the dynamical time scale of interest in the present work is much longer than the time scale of ionisation and recombination.

The momentum equations for the electrons and ions are

$$0 = -q_j n_j \left(\mathbf{E}' + \frac{\mathbf{v}_j \times \mathbf{B}}{c} \right) - \rho_j \nu_{jn} \mathbf{v}_j. \tag{8}$$

Here, $q_j n_j (\mathbf{E}' + \mathbf{v}_j \times \mathbf{B}/c)$ is the Lorentz force where $\mathbf{E}' = \mathbf{E} + \mathbf{v}_n \times \mathbf{B}/c$ is the electric field in the neutral frame with $\mathbf{E}$ and $\mathbf{B}$ as the electric and magnetic fields, respectively, n_j is the number density, $q_e = -e$ for electrons and $q_i = e$ for ions, and c is the speed of light. The momentum equations for the negatively charged (subscript d) and neutral (subscript $d0$) dust grains are

$$0 = -e\, n_d \left(\mathbf{E}' + \frac{\mathbf{v}_d \times \mathbf{B}}{c} \right) - \rho_d \nu_{dn} \mathbf{v}_d - \rho_d \nu^*_{di} (\mathbf{v}_d - \mathbf{v}_{d0}), \tag{9}$$

$$0 = -\rho_{d0} \nu_{d0n} \mathbf{v}_{d0} + \rho_{d0} \nu^*_{d0e} (\mathbf{v}_d - \mathbf{v}_{d0}). \tag{10}$$

The neutral momentum equation is

$$\rho_n \frac{d\mathbf{v}_n}{dt} = -\nabla P + \sum_{e,i,d} \rho_j \nu_{jn} \mathbf{v}_j, \tag{11}$$

where P is the neutral pressure. The inertia and pressure gradient terms have been neglected in Eqs. (8)–(10) since we shall consider low frequency fluctuations in a weakly ionized medium where $n_d < n_e \lesssim n_i \ll n_n$.

We shall define the mass density of the bulk fluid as $\rho \approx \rho_n$. Then, the bulk velocity is $\mathbf{u} \approx \mathbf{v}_n$. The continuity equation [summing up Eq. (7)] for the bulk fluid becomes

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho\, \mathbf{u}) = 0. \tag{12}$$

The momentum equation can be derived by adding Eqs. (8)–(11)

$$\rho \frac{d\mathbf{u}}{dt} = -\nabla P + \frac{\mathbf{J} \times \mathbf{B}}{c}. \tag{13}$$

The generalized Ohms law $\mathbf{J} = \sigma_\parallel \mathbf{E}_\parallel + \sigma_P \mathbf{E}_\perp + \sigma_H\, \mathbf{E} \times \mathbf{B}/B$ can be derived from Eqs. (8) and (9). Inverting generalized Ohms law in terms of electric field, $\mathbf{E}$ gives

$$\frac{c^2}{4\pi} \mathbf{E}' = \eta\, \mathbf{J}_\parallel + \eta_H\, \mathbf{J} \times \mathbf{b} + \eta_\mathbf{P}\, \mathbf{J}_\perp. \tag{14}$$

Here, $\mathbf{b} = \mathbf{B}/|\mathbf{B}|$, and the electric field is written in the neutral frame and parallel and perpendicular components of the current $\mathbf{J}$ refers to the orientation with respect to the ambient magnetic field. The Pedersen diffusivity $\eta_P = \eta + \eta_A$. The Ohm (η), Hall (η_H), and ambipolar (η_A) diffusivies are

$$\eta = \frac{c^2}{4\,\pi \sigma_\parallel},\ \eta_A = \frac{c^2}{4\,\pi} \left(\frac{\sigma_P}{\sigma_\perp^2} - \frac{1}{\sigma_\parallel} \right),\ \eta_H = -\frac{c^2}{4\pi} \frac{\sigma_H}{\sigma_\perp^2}, \tag{15}$$

$$\sigma_\perp = \sqrt{\sigma_P^2 + \sigma_H^2}.$$

The ratio of the cyclotron $\omega_{cj} = q_j B/m_j c$ to the collision ν_{jn} frequency can be defined as plasma Hall parameter

$$\beta_j = \frac{\omega_{cj}}{\nu_{jn}}, \tag{16}$$

which quantifies the relative drift of the magnetic field through the plasma. Note that at a height of 80–100 km, since $\omega_{ce} \sim 10^7\ \mathrm{s}^{-1}$, $\omega_{ci} \sim 10^2\ \mathrm{s}^{-1}$, and, $\omega_{cd} \sim 10^{-3}\ \mathrm{s}^{-1}$ from the collision frequencies, Eqs. (2) and (3), one gets $\beta_e \gg 1$ and $\beta_g \ll \beta_i \ll 1$. Thus, taking curl of Eq. (14) and using the Maxwells equation $c \nabla \times \mathbf{E} = -\partial_t \mathbf{B}$, in $\beta_e \gg 1$ and $\beta_g \ll \beta_i \ll 1$ limit, we arrive at the following induction equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left[(\mathbf{u} \times \mathbf{B}) + \beta_i^2 \frac{(\mathbf{J} \times \mathbf{B}) \times \mathbf{B}}{c\, \rho_i\, \nu_{in}} - \beta_i^2 \frac{\mathbf{J} \times \mathbf{B}}{e\, n_i} - \frac{4\,\pi\,\eta}{c} \mathbf{J} \right]. \tag{17}$$

Not all terms on the right hand side in the preceding equation is important in the mesosphere. The ratio of the Hall to the convective terms is

$$\frac{\beta_i^2 |\mathbf{J} \times \mathbf{B}|/e n_i}{|\mathbf{u} \times \mathbf{B}|} \sim \beta_i^2 \frac{\rho_n}{\rho} \frac{v_A}{L \omega_{ci}}. \tag{18}$$

Thus, for the density ratio $\sim 10^{10}$, $v_A \sim 0.04$ km/s, and $\beta_i \sim 10^{-2}$, above estimates suggest that Hall will dominate the convective term over $L \sim 10^3$ km. The ratio of the ambipolar and Hall term gives

$$\frac{|\mathbf{J} \times \mathbf{B} \times \mathbf{B}|/c\, \rho_i\, \nu_{in}}{|\mathbf{J} \times \mathbf{B}|/e\, n_i} \sim \beta_i \tag{19}$$

and thus Hall will dominate the ambipolar term. The ratio of Hall and Ohm term gives

$$\frac{|\mathbf{J} \times \mathbf{B}|/e\, n_i}{4\,\pi\, \eta |\mathbf{J}|/c} \sim \beta_e, \tag{20}$$

and thus, Hall dominates Ohm. Clearly, Hall is the dominant diffusion mechanism of the magnetic field at 85–100 km.

III. THE DISPERSION RELATION

We assume that the plasma is immersed in a uniform background magnetic field $\mathbf{B}=(0,0,B)$. Often, plasma inhomogeneities coupled with the field aligned current are thought to be responsible for wide range of waves and fluctuations in Earth's ionosphere. The field aligned currents have been observed in the past.[29] Such a current could be caused by the relative plasma motion. Although the role of dust in the field aligned current in Earth's magnetosphere-ionosphere coupling is not clear, in the Jovian and Saturn's magnetosphere-ionosphere coupling, dust will play an important role. In the laboratory plasmas, the presence of field aligned current may cause the lowering of threshold for lower hybrid waves.[30] We shall assume an equilibrium current $\mathbf{J}=(0,0,J)$ aligned to the ambient magnetic field. Although, we have investigated the stability of such an equilibrium in a dusty plasma,[31] here, the medium is predominantly neutral which through collisional coupling to the plasma causes the drift of the magnetic field through the medium. Since Ohm diffusion mainly causes dissipation of the waves, we shall neglect Ohm and retain only Hall and ambipolar diffusion in the induction Eq. (17) for further analysis.

Linearizing, Eqs. (12), (13), and (17) around the equilibrium and assuming that the waves are propagating along the background magnetic field, i.e., along z, after Fourier transforming the perturbed quantities as $exp(-i\omega t+ikz)$, following dispersion relation can be derived from above equations

$$\left(\frac{\omega}{\omega_A}\right)^4 - C_3\left(\frac{\omega}{\omega_A}\right)^3 - C_2\left(\frac{\omega}{\omega_A}\right)^2 + C_1\left(\frac{\omega}{\omega_A}\right) + C_0 = 0, \quad (21)$$

where $\omega_A = k\nu_A$ is the Alfvén frequency and

$$C_0 = 1-\left(\frac{4\pi J}{c\,k\,B}\right)^2, C_3 = 2\beta_i^2\left[\frac{J}{e\,n_i\,v_A}+i\frac{\omega_A}{\nu_{ni}}\right],$$
$$C_2 = 2+\beta_i^4\left[\left(\frac{\omega_A}{\nu_{ni}}-i\frac{J}{e\,n_i\,v_A}\right)^2+\left(\frac{\omega_A}{\omega_H}+i\,\beta_i\frac{J}{e\,n_i\,v_A}\right)^2\right],$$
$$C_1 = 2\,i\,\beta_i^2\left[\frac{\omega_A}{\nu_{ni}}-\frac{J}{e\,n_i\,v_A}\left(1-i\,\beta_i\frac{4\pi J}{c\,k\,B}\right)+i\frac{4\pi J}{c\,k\,B}\frac{\omega_A}{\omega_H}\right]. \quad (22)$$

Here, we have defined the Hall frequency

$$\omega_H = \frac{\rho_i}{\rho_n}\omega_{ci}. \quad (23)$$

It is clear from Eq. (21) that when $J=0$, the dispersion relation in $\beta_i \ll 1$ limit describes the Alfvén wave. Only when Hall effect is present, the dispersion relation provides following roots:

$$\left(\frac{\omega}{\omega_A}\right)_{1,2} = \mp\beta_i^2\frac{\omega_A}{\omega_H}+\left[1+\frac{1}{4}\beta_i^4\left(\frac{\omega_A^2}{\omega_H}\right)^2\right]. \quad (24)$$

The preceding equation describes familiar Alfvén wave in $\omega_A \ll \omega_H$ limit and right circularly polarised whistler (plus sign) and left circularly polarised (negative sign) ion–cyclotron wave in $\omega \ll \omega_A$ limit. The cut-off of the left circular ion cyclotron wave is

$$\omega_{\text{cut-off}} = \beta_i^{-2}\,\omega_H, \quad (25)$$

which is dependent on the ion magnetization (β_i) as well as on the ration of ρ_i/ρ_n that enters in the above expression through Hall frequency. Since $\beta_i \sim 10^{-2}$ and $\rho_i/\rho_n \sim 10^{-10}$, the cutoff frequency is quite small and thus it would appear that both whistler as well as ion-cyclotron waves can easily propagate in the mesosphere.

When $J \neq 0$, the system can become easily unstable and the necessary condition for the instability $C_0 < 0$ reads

$$J > \frac{c\,B}{4\pi}k, \quad (26)$$

which implies that no matter how weak the strength of the ambient current density, this condition can be always satisfied for sufficiently long wavelength fluctuations. Therefore, we may conclude that low frequency fluctuations in the medium will be suspect to the current driven instability.

Writing $\omega=\omega_r+i\omega_i$, the growth rate of the instability in small β_i limit becomes

$$\omega_i = \left(\frac{4\pi J/c}{k\,B}-1\right)^{1/2}\omega_A. \quad (27)$$

Note that the expression in the bracket on the right hand side is always positive for the instability to exist. The growth rate, Eq. (27) can be also be written as

$$\omega_i = \left(\frac{J/e\,n_e\,v_A}{k\,\delta_i}-1\right)^{1/2}\omega_A, \quad (28)$$

where ion skin depth $\delta_i = v_A/\omega_{ci}$.

IV. DISCUSSION AND SUMMARY

We solve Eq. (21) numerically. The growth rate of the instability ω_i/ω_A against $4\pi J/c\,B\,k$ is shown in Fig. 1 for fixed $J/(e\,n_e)=0.1\nu_A$, $\beta_i=10^{-2}$, and $\omega_A/\nu_{ni}=0$ and 10^3. For lower mesosphere, the neutral density density is $\sim 10^{14}\,\text{cm}^{-3}$ and for $m_n = 2.33\,m_p$, where m_p is proton mass, Alfvén velocity in the neutral medium becomes $v_A \sim 0.04\,\text{km/s}$. Thus, $0.1\,\nu_A$ implies plasma drift velocity $J/(e\,n_e) \sim 4\,\text{m/s}$.

The term ω_A/ν_{ni} in the dispersion relation is due to the ambipolar term in the induction equation. Thus, $\omega_A/\nu_{ni}=0$ correspond to purely Hall case whereas $\omega_A/\nu_{ni}=10^3$ implies that ambipolar term dominates Hall term. It appears from the Fig. 1 that the growth rate is insensitive to the diffusive mechanism that is operating in the plasma since growth rate hardly changes when ω_A/ν_{ni} is changed by three order of magnitude.

One can estimate typical current density required to satisfy Eq. (26) in the mesosphere. To that end, assuming that the fluctuation wavelength $\lambda \lesssim 5\text{km}$, for $B=0.3$ Gauss, we get $J \gtrsim 0.03\text{A/m}^2$. We can estimate the electric field that is required for such a current. For plasma conductivity,

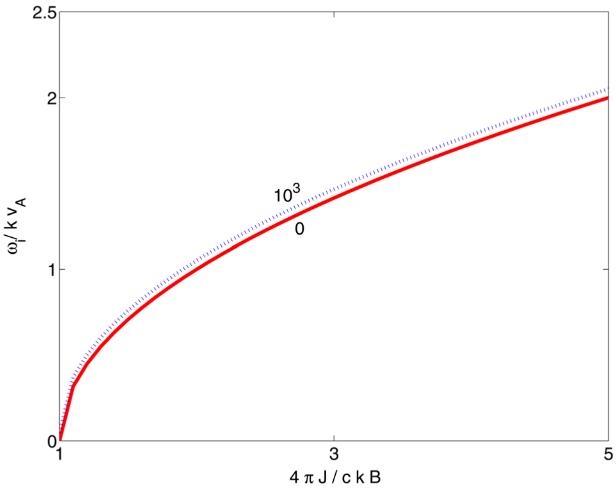

FIG. 1. (Color online) The growth rate ω_i/ω_A against $4\,\pi J/c\,B\,k$ is shown in the figure for $J/(e\,n_e)=0.1\nu_A$, $\beta_i=0.01$, $\omega_A/\nu_{ni}=0$, and 10^3.

$\sigma=n_e\,e^2/m_e\,\nu_{en}\sim 10^7\,\mathrm{s}^{-1}$ from $J=\sigma\,E$, one gets $E\sim 1$ V/m. Therefore, present current driven instability can operate in the mesosphere, only if at least 1 V/m or larger field exists in the NLC layer. As has been noted in the introduction, the electric field measurement has not yet produced unambiguous data. Although there is no general consensus about the field strength, it is quite likely that field is weaker than 1 V/m. If the electric field in the region is $\sim$.01 V/m as some observations suggest,[1] then current in the layer is much weaker than needed for the onset of this instability. As a result, the dusty layer will remain unaffected by the low frequency fluctuations. However, 3 V/m in the NLC without PMSE[14] will destabilise the NLC layer. The observations between 90–130 km shows the presence of a very weak current in sporadic E region.[15] Clearly, this instability can not operate between 90–130 km.

Does Hall or ambipolar diffusion have any role in exciting this current driven instability? To answer this question, we set both Hall and ambipolar terms to zero. The dispersion relation becomes

$$\left(\frac{\omega}{\omega_A}\right)^4-2\left(\frac{\omega}{\omega_A}\right)^2+1-\left(\frac{4\,\pi\,J}{c\,k\,B}\right)^2=0, \tag{29}$$

which leads to same necessary condition and growth rate as described above. As can be seen from Eq. (28), maximum growth rate of the instability corresponds to $k\delta_i \ll 1$, which implies that it is the usual MHD instability in an overwhelmingly neutral medium where neutral inertia balances the bending of the field. Therefore, Hall and ambipolar diffusion do not play any direct role in this instability.

Recall that dusty mesospheric layer fall just below the electrojet region and thus, the region could as well be subject to Farley-Buneman instability. Therefore, it will be of interest to compare the growth rate due to current driven instability, Eq. (28) with the Farley-Buneman instability which is believed to be behind the electrojet. The growth rate γ of Farley-Buneman instability in a weakly ionised, cold medium is[32]

$$\gamma \approx \frac{\beta_i^2\,\beta_e^2(\mathbf{k}\cdot\mathbf{u})^2}{\nu_{in}(1+\beta_i\,\beta_e)^3}\left[1-\beta_i^2\right]. \tag{30}$$

In the summer polar mesosphere, the neutral velocity $\sim$10 cm/s has been observed.[11] Thus, for $\beta_e\sim 10^3$ and $\beta_i\sim 10^{-2}$ and wavelength $\lambda \lesssim 5$km, the growth rate of Farley-Buneman instability is very small ($\gamma \ll 1$). In comparison, the growth rate of current driven instability, Eq. (28), gives $\omega_i \sim k\,v_A \gtrsim 0.05$Hz. Therefore, in the dusty layer of the mesosphere, Farley-Buneman is unimportant. However, the current driven instability requires very large current and it is unclear if such a current exists in the mesosphere.

To summarise, the current driven instability in a collisional, magnetized, dusty medium has been analysed in the present work. The low frequency, long wavelength waves in the magnetic field aligned current medium can become unstable to the low frequency fluctuations if the ratio of the current to the background field is larger than the light speed times the wavenumber. The growth rate of the instability depends on $J/e\,n_e\,v_A$. The linear stability analysis suggests that the dusty mesospheric layer could become unstable provided an electric field $\gtrsim 1$V/m is present in the layer. Although such a strong field may not exist in the medium, there is no unambiguous picture about the field strength until now.[1]

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