Opportunistic Routing in Wireless Ad Hoc Networks: Upper Bounds for the Packet Propagation Speed

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***Abstract*—Classical routing strategies for mobile ad hoc networks forward packets on a pre-defined route (typically obtained by a shortest path routing protocol). Recent research has highlighted the interest in developing opportunistic routing schemes, where the next relay is selected dynamically for each packet and each hop. This allows each packet to take advantage of the local pattern of transmissions at any time. The objective of such opportunistic routing schemes is to minimize the end-to-end delay required to carry a packet from the source to the destination.**

In this paper, we provide upper bounds on the packet propagation speed for opportunistic routing, in a realistic network model where link conditions are variable. We analyze the performance of various opportunistic routing strategies and we compare them with classical routing schemes. The analysis and simulations show that opportunistic routing performs significantly better. We also investigate the effects of mobility. Finally, we present numerical simulations that confirm the accuracy of our bounds.

I. Introduction

In conventional routing strategies for mobile ad hoc networks, packets follow the same route from a source to a destination. The route is usually obtained by a shortest path routing protocol, such as OLSR [5] or AODV [10]. Recent research has highlighted the interest in developing opportunistic routing schemes, where the next relay is selected dynamically for each packet and each hop. As a result, each packet can take advantage of the local pattern of transmissions at each hop and at any time. The general aim of such opportunistic routing schemes is to minimize the end-to-end delay required to carry a packet from the source to the destination.

Several strategies have been proposed, based on geographic routing and/or time-space opportunistic routing. Geographic routing strategies [4] use the positions of the nodes to determine the route to the destination, while they try to optimize geographic criteria, such as the distance to the destination. In time-space opportunistic routing [2], [3], the selection of each relay takes advantage not only of the local topology but also of the current MAC and channel conditions. However, performance evaluations are often limited to comparative simulations (*e.g.*, [2], [4]) or measurements (*e.g.*, [3]) as a complete understanding of what is required for optimal performance (*e.g.*, through theoretical bounds) is still lacking. In this context, our objective is to evaluate the maximum speed at which a packet of information can propagate with opportunistic routing in a multi-hop wireless network. In terms of related work on analytical propagation speed bounds, the authors of [9] used a unit disk network model to calculate an upper bound on the information propagation speed achievable in delay tolerant networks. In the present paper, our contributions are the following:

- we upper-bound the optimal performance, in terms of delay, that can be achieved using any opportunistic routing algorithm, in a realistic network and interference model; we also investigate the effect of mobility;
- we verify the accuracy of our bounds using numerical simulations: we compare them with the performance of an optimized time-space opportunistic routing scheme [2];
- we also compare opportunistic and classical (*i.e.*, shortest path based) routing; the analysis and the simulations show that opportunistic routing performs significantly better.

II. Model and classical routing

A. Network and propagation model

Using the model developed in [1], we consider a network on an infinite 2-D map, with a constant density of ν nodes per square area unit, dispatched according to a Poisson distribution. We assume that time is slotted, and at each slot, each node has a packet to transmit with probability $\frac{\lambda}{\nu}$, with $\lambda < \nu$. Therefore, the distribution of the number of active transmitters per slot is Poisson; the rate of transmitters per square area unit and per slot is λ. We assume that all nodes transmit at the same nominal power. We take a simple power attenuation function, with the attenuation coefficient $\alpha > 2$: the signal level received at distance r from the transmitter is $W = \frac{\exp(F)}{r^\alpha}$, where F is a random fading of mean 0. A packet can be successfully decoded if its signal-over-noise ratio (SNR) is greater than a threshold K. By noise, we mean the sum of powers received from all other transmissions in the same slot.

Let us denote by $W(\lambda)$ the total power received by a node at a random slot. According to [1], the Laplace transform of $W(\lambda)$ can be calculated exactly, assuming *w.l.o.g.* that all transmitters emit at unit nominal power. The Laplace transform $\tilde{S}(\theta, \lambda) = E(e^{-W(\lambda)\theta})$ has the following expression:

$$\tilde{S}(\theta, \lambda) = \exp(-\lambda\pi\Gamma(1 - \frac{2}{\alpha})E(e^{\frac{2}{\alpha}F})\theta^{\frac{2}{\alpha}}) \qquad (1)$$

We note that the random variable $W(\lambda)$ is invariant by translation, *i.e.*, it does not depend on the location of node.

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For general α, there is no closed formula for the probability function $P(W(\lambda) < x)$. However, we have the following series expansion and asymptotic behavior [8] (we denote $\gamma = \frac{2}{\alpha}$ and $C = \pi\Gamma(1-\gamma)E(e^{\gamma F})$):

- $P(W(\lambda) < x) = \sum_{n\geq 0}(-C\lambda)^n \frac{\sin(\pi n\gamma)}{\pi}\frac{\Gamma(n\gamma)}{n!}x^{-n\gamma}$,
- $-\log P(W(\lambda) < x) = \Theta(x^{\frac{\gamma}{\gamma-1}})$ when $x \to 0$.

Therefore, a silent node will correctly receive (with SNR at least K) a packet from another node at distance r with probability $p(r) = P(W(\lambda) < \frac{e^F}{K}r^{-\alpha})$. By substitution in the series expansion, we obtain the probability $p(r)$:

$$p(r) = \sum_{n\geq 0}(-C\lambda)^n \frac{\sin(\pi n\gamma)}{\pi}\frac{\Gamma(n\gamma)}{n!}E(e^{-\gamma F})K^{n\gamma}r^{2n} \quad (2)$$

When we take $\alpha = 4$ (*i.e.,* in the case corresponding to the reflection-absorption model of wave propagation over an infinite plane), we obtain the following closed formula:

$$p(r) = 1 - E\left(\operatorname{erf}(\frac{1}{2}\lambda\pi^{\frac{3}{2}}K^{\frac{1}{2}}e^{-\frac{F}{2}}r^2)\right),$$

where erf() is the error function; the expectation $E(.)$ indicates the average with respect to the random fading factor F.

B. Classical routing bound

Based on the model and the previous analysis, we can establish a first upper bound on the packet propagation speed, when a classical routing strategy is employed, *i.e.,* when all packets from a source to a destination follow the same route. The propagation delay is caused by the fact that packets must be retransmitted several times until a correct reception occurs.

Theorem 1: In classical routing the packet propagation speed is bounded from above by the quantity $(1 - \frac{\lambda}{\nu})\max_{r\geq 0}\{rp(r)\}$.

Proof: We assume *w.l.o.g.* that the source is at position 0 and the destination at position $\mathbf{z}$. Let us suppose that the chain of relays between the source and the destination is made up of n nodes. We denote by $\mathbf{z}_i$ the position of node i, with $\mathbf{z}_1 = 0$ and $\mathbf{z}_n = \mathbf{z}$. The probability of correct reception between node i and node $i+1$ is $p(|\mathbf{z}_{i+1} - \mathbf{z}_i|)(1 - \frac{\lambda}{\nu})$; the term $(1 - \frac{\lambda}{\nu})$ corresponds to the probability that the receiver is idle (*i.e.,* it is not transmitting simultaneously). Therefore, the delay for a correct transmission is on average $\left(p(|\mathbf{z}_{i+1} - \mathbf{z}_i|)(1 - \frac{\lambda}{\nu})\right)^{-1}$. As a result, the average packet speed on the path is: $(1 - \frac{\lambda}{\nu})\frac{|\mathbf{z}|}{\sum_i \frac{1}{p(|\mathbf{z}_{i+1} - \mathbf{z}_i|)}}$. This quantity is smaller than $(1 - \frac{\lambda}{\nu})\frac{\sum_i |\mathbf{z}_{i+1} - \mathbf{z}_i|}{\sum_i \frac{1}{p(|\mathbf{z}_{i+1} - \mathbf{z}_i|)}}$ (from the triangle inequality), which in turn is smaller than $(1 - \frac{\lambda}{\nu})\max_{r\geq 0}\{rp(r)\}$. ■

III. Methodology

As we discussed in the previous section, in classical routing, each node selects as the next relay the node that offers the best compromise between its routing delay towards the destination and its probability of receiving the packet, and all packets follow the same route. On the other hand, opportunistic routing consists in selecting the best chain of relays in terms of actual delay (for each hop and each slot).

In wireless networks, the reception quality of a signal can vary greatly due to the variation in the Signal-over-Noise Ratio (SNR). These variations provide substantial possibilities for improving routing performance when opportunistic strategies are employed. In the following sections, we evaluate the maximum speed at which a piece of information can be propagated in the network with opportunistic routing, and compare this with classical routing. We establish generic upper bounds, but do not analyze specific algorithms; this is achieved because our analysis unfolds all possible paths. In terms of algorithms, our upper-bound would be attainable if all SNR variations in the network were known in advance.

We investigate the performance of two generic strategies, which we call "store-forward", and "store-hold-forward". In the first strategy, nodes attempt to forward packets immediately. The second strategy is more general: each relay has the choice of either immediately transmitting the packet or waiting for better signal propagation conditions. We decompose paths into segments and we use language theory to evaluate the Laplace transforms of the path probability density. From the Laplace transforms and complex analysis based on the saddle point method, we establish an upper bound on the average number of paths arriving at a point $\mathbf{z}$ before a time t, where $\mathbf{z}$ is a 2D space vector. Using this approach, we prove our theorems and theoretical bounds on the packet propagation speed.

Path probability density and Laplace transform

Formally, a path is a space-time trajectory of the packet between the source and the destination. We assume that time zero is when the source transmits, and we will check at what time t the packet is received at coordinate $\mathbf{z} = (x, y)$. We will only consider simple paths, *i.e.,* paths which never return twice through the same node. As we will discuss in Section IV, this does not affect the validity of our final results.

Let $\mathcal{C}$ be a simple path. Let $Z(\mathcal{C})$ be the terminal point. Let $T(\mathcal{C})$ be the time at which the path terminates. Let $p(\mathcal{C})$ be the probability of path $\mathcal{C}$. In the following, we will in fact consider a path as a discrete event in a continuous set and, therefore, the probability weight should be converted into a probability density. We call $p(\mathbf{z}, t)$ the average number of paths that arrive at $\mathbf{z}$ before time t:

$$p(\mathbf{z}, t) = \lim_{r\to 0}\frac{1}{\pi r^2}\sum_{|\mathbf{z}-Z(\mathcal{C})|<r, T(\mathcal{C})<t} p(\mathcal{C}) \ .$$

We now express the probability $q(\mathbf{z}, t)$ that there exists at least one path that arrives *at the destination node* before time t ($p(\mathbf{z}, t)$ is not conditioned on the existence of a node at $\mathbf{z}$).

Lemma 1: The probability $q(\mathbf{z}, t)$ that there exists at least one path that arrives at a destination node, located at $\mathbf{z}$, before time t, satisfies: $q(\mathbf{z}, t) \leq A\int_0^t p(\mathbf{z}, t)dt$, where $p(\mathbf{z}, t)$ is the probability density of paths and A is a finite positive number.

Proof: Quantity $p(\mathbf{z}, t)$ is the density of paths starting from the origin at time 0 and ending on $\mathbf{z}$ at time t. Therefore, the average number of paths starting at $(0, 0)$ and ending in a space area B at time t is $\int_B p(\mathbf{z}, t)d\mathbf{z}dt$, with the integral being multi-dimensional. The number $n(\mathbf{z},t)$ of paths that start on $(0,0)$ and arrive on a node at point $\mathbf{z}$ at time t is exactly:

$$\begin{array}{rcl} n(\mathbf{z},t) & = & \int d\mathbf{z}' p(|\mathbf{z}'-\mathbf{z}|)p(\mathbf{z}',t-1) \\ & \leq & Ap(\mathbf{z},t-1), \end{array}$$

with $A \geq \int_0^\infty e^{ar}p(r)2\pi r dr$.

Notice that A is finite if $p(\mathbf{z},t)$ grows at most exponentially, *i.e.,* if $\frac{p(\mathbf{z}',t)}{p(\mathbf{z},t)} \leq e^{a|\mathbf{z}'-\mathbf{z}|}$ (from (2) in Section II, we already know that $p(r)$ has a super-exponential decay). This condition is true, as is shown in Section IV (see Lemma 3).

Let $q(\mathbf{z},t)$ be the probability that there exists a path that arrives at point $\mathbf{z}$ before time t. We have $q(\mathbf{z},t) \leq N(\mathbf{z},t)$ where $N(\mathbf{z},t)$ is the average number of paths that end at node $\mathbf{z}$ before time t. We have $q(\mathbf{z},t) \leq A\int_0^t p(\mathbf{z},t)dt$. ∎

In the next sections, we calculate when $q(\mathbf{z},t)$ becomes 0 almost surely; this analysis provides our upper bounds on the packet propagation speed. For the calculations, we make use of Laplace transforms. Let ζ be a space vector and θ a scalar. We denote by $w(\zeta,\theta)$ the path Laplace transform:

$$w(\zeta,\theta) = E(\exp(-\zeta \cdot Z(\mathcal{C}) - \theta T(\mathcal{C})))$$

defined for a domain definition for (ζ,θ). Notice that $\zeta \cdot Z(\mathcal{C})$ is the dot product of two space vectors.

In the following, we split the path into segments $\mathcal{C} = (s_1, s_2, \ldots, s_k)$, such that $p(\mathcal{C}) = p(s_1)p(s_2)\cdots p(s_k)$. Note that each segment is a space-time vector. Based on the decomposition, we can compute the Laplace transform of the path, using the Laplace transforms of the individual segments. For this example, the path $\mathcal{C}$ is described as a Cartesian product of the segments $s_1, s_2, \ldots$; therefore, the Laplace transform of the path $\mathcal{C}$ can be expressed as the product of the Laplace transforms of the segments. Equivalently, a union (*i.e.,* a choice to use one *OR* another segment to obtain the path) translates into a sum of Laplace transforms. If we express a path as an arbitrary sequence of the same type of segments s (*i.e.,* using regular expression notation: $\mathcal{C} = s^*$), the path Laplace transform has the expression: $w(\zeta,\theta) = \frac{1}{1-l(\zeta,\theta)}$, where $l(\zeta,\theta)$ is the Laplace transform of segments s. This is the equivalent of the formal identity $\frac{1}{1-y} = 1 + y + y^2 + y^3 + \ldots$, which represents the Laplace transform of an arbitrary sequence of random variables with Laplace transform y. More generally, we can use notation from language theory to express a path with any regular expression which characterizes all permitted combinations of different types of segments. Again, we can automatically deduce the Laplace transform of the path, based on the expressions for individual segments, and using the above constructions/translations (see [7]).

IV. Opportunistic Routing

We first develop our methodology in the simplified framework of the store-forward strategy; then we apply the same techniques to prove our main theorem on the packet propagation speed with the more general store-hold-forward strategy.

A. Opportunistic Store-forward

This is the most basic opportunistic routing, since nodes always attempt to transmit the packets immediately.

a) Routing path segmentation: The routing path is made up only of "emission segments". In other words, a routing path is a sequence of emission segments taken from the alphabet $\{s_e\}$ and is of the form s_e^*. An emission segment s_e is a space time vector; the time component corresponds to the duration of one slot and the space component describes the distance traveled by the packet in one emission. For instance, according to the model in Section II, we can calculate the probability of such a segment as a function of this distance.

Our aim is to find the quickest path that arrives at a given destination at coordinate $\mathbf{z} = (x,y)$. We stress again that we consider only simple paths, *i.e.,* paths that never loop on the same node. However, in a store and forward strategy, a simple path may not be the quickest path that arrives at the destination $\mathbf{z}$, since the quickest path may in fact loop on a node A (potentially encountering more favorable transmission conditions). In an equivalent simple path, node A would need to hold the packet for a certain time before retransmitting it. In any case, the equivalence between quickest path and quickest simple path will be true for the next more interesting strategy: store-hold-forward. The store-forward strategy is only developed as an introduction to the methodology.

b) Path Laplace transform: Let $w(\zeta,\theta)$ be the path Laplace transform, *i.e.,* the Laplace transform of $p(\mathbf{z},t)$:

$$w(\zeta,\theta) = E(p(\mathbf{z},t)\exp(\zeta \cdot \mathbf{z} - \theta t)).$$

In the following lemma, we compute $w(\zeta,\theta)$.

Lemma 2: In the store-forward strategy, the path Laplace transform has the expression:

$$w(\zeta,\theta) = \frac{1}{1-(\nu-\lambda)\Psi_p(|\zeta|)e^{-\theta}},$$

where $\Psi_p(x) = 2\pi\int_0^\infty p(r)I_0(xr)rdr$, and $I_0()$ is a modified Bessel function of order 0.

Notice that $I_0(x) = \sum_{k\geq 0}(\frac{x}{2})^{2k}\frac{1}{(k!)^2}$. Developing with the expression for $p(r)$ in (2), we get:

$$\Psi_p(\rho) = \pi\sum_{k\geq 0}\frac{1}{\gamma\Gamma((k+1)\gamma)k!}\frac{E(e^{(k+1)\gamma F})}{(CK^\gamma)^{k+1}}\left(\frac{\rho}{2}\right)^{2k}$$

We also note that, when we take $F=0$, $K=1$ and $\alpha=4$, we have the specific formula:

$$\Psi_p(\rho) = \frac{2}{\pi}H\left([\frac{1}{2},\frac{1}{2}],\frac{\rho^4}{64\pi^3}\right) + \frac{\rho^2}{2\pi^2}H\left([1,\frac{3}{2}],\frac{\rho^4}{64\pi^3}\right),$$

where $H([p,q],x)$ are hypergeometric functions.

Proof: In the store-forward model, a path is only made up of successful emission segments s_e. An emission segment is a space time vector $(\mathbf{z},1)$ where $\mathbf{z}$ is a space vector and we assume that 1 is the slot time unit. An emission segment is successful if it ends on a mobile node (with density ν), if the receiver is not transmitting simultaneously (with probability $1-\frac{\lambda}{\nu}$) and if the transmission is successful (with probability $p(|\mathbf{z}|)$). Therefore, the density probability of emission segments is $p(|\mathbf{z}|)\nu(1-\frac{\lambda}{\nu})$ in space (which corresponds to the previously stated conditions) and is a Dirac measure on 1 in time (*i.e.,* the duration is always one slot).

We denote the space vector $\mathbf{z} = (r\cos\phi, r\sin\phi)$, where r is the segment length and $\phi \in [0,2\pi]$ is the direction. Since our analysis unfolds all possible paths, emission segments only depend on the nodes' positions. Therefore, they can be considered independent and the direction angle ϕ of each segment is uniformly distributed in $[0, 2\pi]$. The emission segment Laplace transform $g_e(\zeta, \theta) = E(\exp(-\zeta \cdot \mathbf{z} - \theta))$ is obtained by averaging on r and ϕ:

$$\begin{aligned} g_e(\zeta,\theta) &= e^{-\theta}\int_0^{\infty}\nu(1-\frac{\lambda}{\nu})p(r)rdr\int_0^{2\pi}e^{-|\zeta|\cdot r\cos\phi}d\phi \\ &= 2\pi e^{-\theta}(\nu-\lambda)\int_0^{\infty}p(r)I_0(|\zeta|r)rdr. \end{aligned}$$

Since the path is equivalent to a sequence emission segment, expressed as s_c^* with the language wording (see Section III), we have: $w(\zeta, \theta) = \frac{1}{1-g_e(\zeta,\theta)}$. ■

c) Maximum propagation speed: Recall that $q(\mathbf{z}, t)$ is the probability that there exists at least one path that arrives at the destination node before time t. We will prove that $q(\mathbf{z}, t) = O\left(\exp(-\rho_0|\mathbf{z}| + \theta_0 t)\right)$, for some (ρ_0, θ_0).

This implies that $q(\mathbf{z}, t)$ vanishes very quickly when t is smaller than the value such that $-\rho_0|\mathbf{z}| + \theta_0 t = 0$, *i.e.* when $\frac{z}{t} = \frac{\theta_0}{\rho_0}$. Therefore (as shown in [9]), quantity $\frac{\theta_0}{\rho_0}$ is an asymptotic propagation speed upper-bound; for all $v > \frac{\theta_0}{\rho_o}$:

$$\lim_{|\mathbf{z}|\to\infty} q(\mathbf{z}, \frac{|\mathbf{z}|}{v}) = 0$$

Let $D(\rho, \theta) = (\nu - \lambda)\Psi_p(\rho)e^{-\theta}$. We notice that the path Laplace transform is in denominator $1 - D(|\zeta|, \theta)$. The key of the analysis is the set $\mathcal{K}$ of pairs (ρ, θ) such that $D(\rho, \theta) = 1$, called the *Kernel*. We show that such a path Laplace transform implies the following asymptotic estimate of path density.

Lemma 3: When $|\mathbf{z}|$ and t both tend to infinity we have: $p(\mathbf{z}, t) = O\left(\exp(-\rho_0|\mathbf{z}| + \theta_0 t)\right)$, where (ρ_0, θ_0) is the element of the kernel $\mathcal{K}$ that minimizes $-\rho|\mathbf{z}| + \theta t$.

Proof: The proof is a direct adaptation of the saddle point analysis in [9], Theorem 2. ■

We can now prove the following theorem concerning the maximum packet propagation speed.

Theorem 2: In the store-forward strategy, the packet propagation speed is upper-bounded by the smallest ratio $\frac{\theta}{\rho}$ of the elements of $\mathcal{K} = \{(\rho, \theta) : D(\rho, \theta) = 1\}$, where: $D(\rho, \theta) = (\nu - \lambda)\Psi_p(\rho)e^{-\theta}$, with $\Psi_p(\rho) = 2\pi \int_0^{\infty} p(r)I_0(\rho r)rdr$, and $I_0()$ is a modified Bessel function of order 0.

Proof: The Kernel of the path Laplace transform is the root of the denominator, *i.e.,* the set of pairs (ρ, θ) such that $D(\rho, \theta) = 1$. Therefore, following the asymptotic analysis of the average number of journeys (from Lemma 3) and Lemma 1, the propagation speed upper bound is given by the minimum ratio $\frac{\theta}{\rho}$ of $(\rho, \theta) \in \mathcal{K}$. ■

B. Store-hold-forward strategy

This strategy differs from the store-forward strategy by the fact that nodes can either transmit the packet immediately after they receive it, or hold it and attempt to transmit later.

Lemma 4: In the store-hold-forward strategy, the path Laplace transform has the expression:

$$w(\zeta, \theta) = \frac{1}{1 - (\nu - \lambda)\Psi_p(|\zeta|)e^{-\theta} - e^{-\theta}}.$$

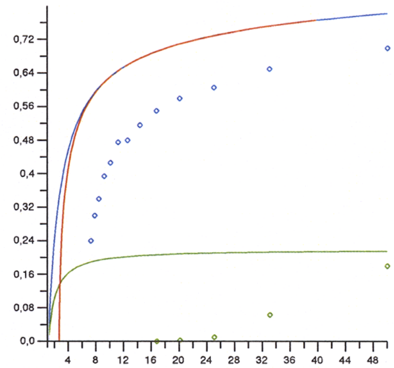

Fig. 1. Store-forward (red), store-hold-forward (blue) packet propagation speed upper-bounds (in meters per slot) versus the node density ν, compared to classical routing (green). The network traffic density is fixed ($\lambda = 1$). The dots correspond to measured values obtained through simulation.

Proof: In this case, a path is made up of an arbitrary sequence of emission and "hold segments". A hold segment is a space-time vector expressing the situation where the packet stays in a node's memory during one slot. Since we assume that all nodes are still, a hold segment is the vector $(0, 1)$, where 0 is the space component and 1 is the slot duration. Hence, the hold segment Laplace transform is $g_h(\zeta, \theta) = e^{-\theta}$.

A path is now a sequence in $\{s_e + s_h\}^*$, since a node can either emit *or* hold the packet. The path Laplace transform is:

$$w(\zeta, \theta) = \frac{1}{1 - (\nu - \lambda)\Psi_p(|\zeta|)e^{-\theta} - e^{-\theta}}.$$

■

Equivalently to Theorem 2, we have the new theorem by substituting the new path Laplace transform.

Theorem 3: In the store-hold-forward strategy, the packet propagation speed is upper-bounded by the smallest ratio $\frac{\theta}{\rho}$ of the elements of $\mathcal{K} = \{(\rho, \theta) : D(\rho, \theta) = 1\}$, where: $D(\rho, \theta) = \left(1 + (\nu - \lambda)\Psi_p(\rho)\right)e^{-\theta}$.

V. Simulations

In this section, we present simulations illustrating the packet propagation speed upper bounds proved in Theorem 1 concerning conventional routing, and Theorems 2 and 3 concerning opportunistic routing strategies.

First, in Figure 1, we plot the theoretical propagation speed bounds as a function of the node density ν, obtained from the theorems when the traffic density is fixed: $\lambda = 1$. The bounds express the maximum speed in meters per slot at which a packet can propagate in the network. The traffic load *per node* equals $\frac{\lambda}{\nu}$, hence as ν increases, the load of the nodes is smaller and the packets can propagate faster. For this numerical example, we take a required signal-to-noise ratio $K = 1$ and a power attenuation coefficient $\alpha = 4$.

Notice that the upper bound for the store-forward strategy collapses to zero speed for a value of $\nu \approx 2.47...$ (below the percolation point, *i.e.*, when the network becomes disconnected). On the other hand, the store-hold-forward strategy

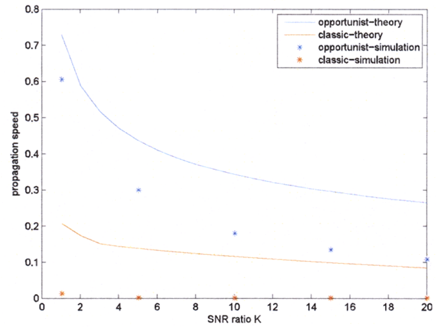

Fig. 2. Propagation speed versus required signal-to-noise ratio K, for $\nu = 25$, for $\alpha = 4$, and $\lambda = 1$. Comparison of theoretical bounds on opportunistic routing (blue) and classical routing (red), as well as simulations (dots).

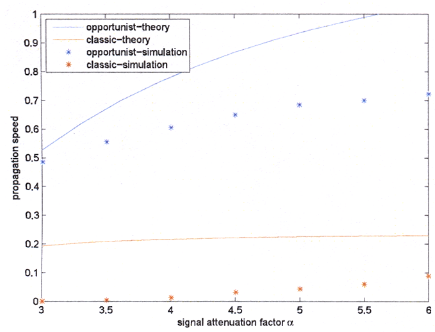

Fig. 3. Propagation speed versus signal attenuation factor α, for $\nu = 25$, $K = 1$, and $\lambda = 1$. Comparison of theoretical bounds on opportunistic routing (blue) and classical routing (red), as well as simulations (dots).

upper bound remains non-zero until the node density ν reaches its minimal value: $\nu = \lambda$ (recall that $\lambda \leq \nu$). This illustrates the fact that the variations in the signal-to-noise ratio always guarantee connectivity, as long as the nodes can hold the packets for the transmission possibilities to change.

In Figure 1, we also compare our theoretical bounds with measured values obtained via simulation of a classical (shortest path based) and an opportunistic routing scheme (dots). We perform the simulations following the framework of [2].

- The *classical routing* strategy is based on a Dijkstra algorithm. We fix a maximum transmission range, which is optimized according to the discussion in Section II-B. The packets are then forwarded following the shortest path (in hops) from the source to the destination.
- We also simulate an *opportunistic routing* algorithm, presented in [2], which is based on time-space opportunistic radial routing. At each hop and each slot, the next relay is the node that is the closest to the destination, among the nodes that capture the packet successfully.

The plots confirm the accuracy of our theoretical bounds on the opportunistic packet propagation speed. They also show that radial time-space routing achieves a close to optimal performance. It is important to note that, in all cases, the opportunistic routing performance is significantly better.

In Figure 2, we illustrate the behavior of the upper bounds and the simulation measurements, for different values of the signal-to-noise ratio K. We fix the node density to $\nu = 25$ and the traffic density to $\lambda = 1$; this means that each node has a packet to transmit with probability $\frac{\lambda}{\nu} = 0.04$ at each slot. We also take $\alpha = 4$. Interestingly enough, the simulated classical routing protocol almost collapses under the given traffic conditions, while opportunistic routing yields a packet propagation speed well above 0. In Figure 3, we plot the theoretical upper bounds and the simulation measurements, for different values of the power attenuation factor α. We fix the node density to $\nu = 25$ and the traffic density to $\lambda = 1$; for the required SNR ratio, we take $K = 1$.

VI. Node mobility

In this section, we adapt the analysis from Section IV to take node mobility into account. The mobility model is the random walk: at each slot a node changes direction with probability τ. We denote by s the speed of the mobile nodes.

A. Path and movement decomposition

Again, we consider a store-hold-forward routing scheme. However, the nodes can now move while they hold the packets. We decompose the path into three kinds of segments:

1) *emission* segments s_e;
2) *move-to-turn* segments s_t;
3) *move-to-emit* segments s_m.

Emission segments are defined in the same manner as in Section IV. The move-to-turn and move-to-emit segments substitute the hold segments. The "move-to-turn" segment corresponds to the straight line that a mobile node follows until it changes direction; it represents one step of the random walk. The "move-to-emit" segment is similar, but now the mobile node emits the packet before the next change of direction; in other words, it corresponds to an incomplete random step.

More precisely, the move-to-turn segment is a space-time vector $(k \cdot \mathbf{u}s, k)$ where $\mathbf{u}$ is a unitary vector (marking the direction of the movement) and k is the number of slots during which the mobile has moved without turning. The segment has a duration of k slots with probability $\tau(1-\tau)^{k-1}$ and $k > 0$. The Laplace transform of a move-to-turn segment is:

$$g_t(\zeta, \theta) = \sum_{k>0} \tau(1-\tau)^{k-1} E(I_0(|\zeta|ks))e^{-k\theta}.$$

The expectation $E(.)$ indicates the average value with respect to the speed factor s.

Similarly, the move-to-emit segment is a space-time vector $(k \cdot \mathbf{u}s, k)$, where k is the number of slots during which the mobile has moved. However, we now have a duration of k slots with probability $(1-\tau)^{k-1}$, since there is no change of direction. This leads to a Laplace transform equal to: $g_m(\zeta, \theta) = \sum_{k>0}(1-\tau)^{k-1}E(I_0(|\zeta|ks)e^{-k\theta})$. Notice that $g_t(\zeta, \theta) = \tau g_m(\zeta, \theta)$.

A path is made up of segments according to the following rules, describing the node movement and the packet transmissions:

1) an s_e segment is followed by any segment;
2) an s_t segment is either followed by an s_t segment or an s_m segment;
3) an s_m segment is always followed by an s_e segment.

Therefore, a path is a word in the alphabet $\{s_e, s_t, s_m\}$, following the regular expression $s_e^*(s_t^* s_m s_e s_e^*)^*(1 + s_t^* s_m)$; this expression decomposes a path according to the three previous rules. According to Section III, we can directly deduce the path Laplace transform from the regular expression:

$$\begin{aligned} w(\zeta,\theta) &= \frac{1}{1-g_e(\zeta,\theta)}\left(1+\frac{g_m(\zeta,\theta)}{1-g_t(\zeta,\theta)}\right)\frac{1}{1-\frac{g_m(\zeta,\theta)}{1-g_t(\zeta,\theta)}\frac{g_e(\zeta,\theta)}{1-g_e(\zeta,\theta)}} \\ &= \frac{1-(1-\tau)g_m(\zeta,\theta)}{1-g_e(\zeta,\theta)-\tau g_m(\zeta,\tau)-(1-\tau)g_m(\zeta,\tau)g_e(\zeta,\theta)} \end{aligned}$$

Notice that, when the speed is $s = 0$, we have $g_m(\zeta, \theta) = \frac{e^{-\theta}}{1-(1-\tau)e^{-\theta}}$, and we find, as expected, the result of the previous section, where there is no mobility. With the new path Laplace transform, we can again apply the saddle point technique, and derive an upper bound on the packet propagation speed.

B. Analysis

We will investigate in detail the realistic case where the speed is small, that is: $s \ll 1$. Indeed, s is expressed in meters per slot, and wireless transmissions are expected to occur faster than physical node motions.

Using the expansion $I_0(\rho) = 1 + \left(\frac{\rho}{2}\right)^2 + O(\rho^4)$, we get:

$$\begin{aligned} g_m(\zeta,\theta) &= \sum_{k>0}(1-\tau)^{k-1}e^{-k\theta}(1+k^2\left(\frac{|\zeta|}{2}\right)^2 E(s^2) \\ &\quad +O(|\zeta|^4 E(s^4)) \\ &= \frac{e^{-\theta}}{1-(1-\tau)e^{-\theta}} + \left(\frac{|\zeta|}{2}\right)^2 \sigma_2 \frac{(1+(1-\tau)e^{-\theta})e^{-\theta}}{(1-(1-\tau)e^{-\theta})^3} \\ &\quad +O(\sigma_4) \end{aligned}$$

where σ_2 and σ_4 are the second and fourth moments of the speed s, respectively.

We take $\rho = |\zeta|$ to simplify the notation. The denominator of the modified path Laplace transform is: $e^\theta - 1 - (\nu - \lambda)\Psi_p(\rho) - g(\rho, \theta)\sigma_2 + O(\sigma_4)$, where:

$$g(\rho,\theta) = (\tau+(1-\tau)(\nu-\lambda)\Psi_p(\rho)e^{-\theta})\left(\frac{\rho}{2}\right)^2\frac{(1+(1-\tau)e^{-\theta})e^{-\theta}}{(1-(1-\tau)e^{-\theta})^2}.$$

Denoting $f(\rho) = 1 + (\nu - \lambda)\Psi_p(\rho)$, we have a kernel set (ρ, θ) such that: $\theta = \log(f(\rho)) + \sigma_2 \frac{g(\theta,\rho)}{f(\rho)} + O(\sigma_2^2 + \sigma_4)$. As a result, when we apply the saddle point analysis of Section IV, the maximum packet propagation speed is:

$$\frac{\log f(\rho_0)}{\rho_0} + \sigma_2 \frac{g(\rho_0, \log f(\rho_0))}{f(\rho_0)\rho_0} + O(\sigma_2^2 + \sigma_4),$$

where ρ_0 is the root of $\frac{f'(\rho)}{f(\rho)} - \frac{\log f(\rho)}{\rho}$.

When $\nu \to \lambda$, we have $f(\rho) \to 1$ and $\theta \to 0$. The residual propagation speed upper bound tends to: $\log\left(1 + (2-\tau)\frac{\rho_0^2}{4}\frac{\sigma_2}{\tau}\right)$, where ρ_0 is the root of $\rho\frac{\Psi_p'(\rho)}{\Psi_p(\rho)} - 1$.

Notice that the propagation speed upper bound is larger than zero, as long as $\frac{\sigma_2}{\tau}$ tends to a positive constant. This is equivalent to saying that the random walk has a constant standard deviation rate per time unit. Conversely, we showed in Section IV that, when the nodes do not move, the propagation speed tends to zero when the node density decreases.

In Figure 4, we plot the difference of the propagation speed bounds when nodes move as opposed to when they stay still. We see that mobility causes a small improvement in the propagation speed. However, this improvement enables the propagation speed to remain larger than zero, even when the node density tends to its minimal value (*i.e.,* $\nu \to \lambda$).

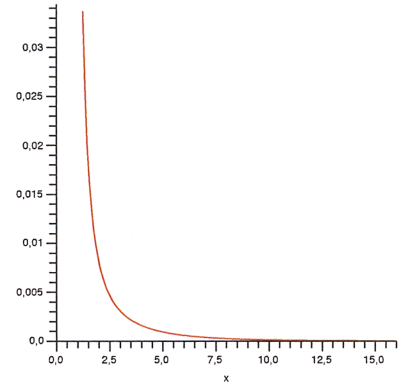

Fig. 4. Improvement in the propagation speed upper bound (with opportunistic routing) versus the node density ν, with traffic density $\lambda = 1$, mobile speed $s = 0.1$ and direction change probability $\tau = 0.1$.

VII. Conclusion

We characterized the optimal performance, in terms of delay, that can be achieved using any opportunistic routing algorithm, in a realistic network model where link conditions are variable. We derived analytical upper bounds on the packet propagation speed with generic opportunistic routing strategies in Theorems 2 and 3. Our analysis is sufficiently general to provide bounds according to the node and traffic densities in the network, as well as the signal propagation conditions. Such theoretical bounds are useful in order to evaluate and/or optimize the performance of specific opportunistic routing algorithms. Furthermore, we compared opportunistic and classical (*i.e.,* shortest path based) routing; we showed that opportunistic routing performs significantly better. We presented numerical simulations that confirm the accuracy of our bounds in numerous scenarios: we simulated a classical routing algorithm and an optimized time-space opportunistic routing scheme. We also showed that opportunistic store-hold-forward schemes can take advantage of the nodes' mobility.

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