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SHORT REPORT

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A new unique impulse response function in linear vector autoregressive models

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Abstract

This article proposes a new unique impulse response function (IRF) measure, or MIRF, based on the popular vector autoregressive model to study interdependency of multivariate time series. Same as the orthogonal IRF, the estimator of MIRF has an analytical form with well-established asymptotics, and is invariant to ordering of series. Compared to alternative unique IRF measures, MIRF does not depend on extreme identifications, and the associated forecast error variance measure is explainable. An illustrative empirical example is also provided.

KEYWORDS

eigendecomposition, forecast error variance decomposition, impulse response function, vector autoregressive

JEL CLASSIFICATION

C32, C58

1 | INTRODUCTION

The popularity of the vector autoregressive (VAR) model starts from the seminal work of Sims (1980), which is a natural extension of the AR model to the multivariate framework. For novel finance practices, VAR is still a methodology to investigate the interdependency of multivariate time series (see, e.g., Caraiani et al. (2021), among others).

Impulse response function (IRF) is one of the most important metrics based on VAR. For instance, together with the associated forecast error variance decomposition (FEVD), IRF may describe how interested finance and/or economic series react over time to exogenous impulses (Da Fonseca & Gottschalk, 2020; Scherrer & Fernandes, 2021). However, as recommended by Sims (1980), interpretable IRF usually depends on orthogonalized errors, and such a measure is known as the orthogonal IRF (OIRF). Since the orthogonalization is executed via the Cholesky decomposition, ordering of series in a VAR system will affect the estimated IRF and FEVD. Specifically, this ordering suggests the recursive casual relationship for contemporaneous values, such that variables ordered in the front can impact all those ranked behind, but not vice versa. Unfortunately, prior knowledge to identify a justifiable ordering does not always exist.

The variance to ordering issue is widely recognized in the literature. As one of the most influential solutions, the generalized IRF (GIRF) proposed by Pesaran and Shin (1998) does not require orthogonal errors and therefore provides a unique measure. Despite its uniqueness, the structure of GIRF actually leads to extreme identifying assumptions which may reduce the reliability of estimation (Kim, 2013). Moreover, the associated FEVD of GIRF does not always add to 100% for each response, which causes difficulty in explanation. In a more recent attempt, Lanne and Luoto (2016) derive a unique identification based on a more complicated Bayesian structural VAR model investigated by Lanne et al. (2017). The related computational intensity, however, is much higher than the usual VAR system, especially when the dimensionality is high, and/or the sample size is large.

In this article, we propose an alternative approach of the Cholesky decomposition to orthogonalize errors. This new measure is named modified IRF (MIRF). We show that MIRF is unique, and its estimator retains all the attractive properties of the OIRF counterpart, such as the existence of an analytical form and well-established asymptotics. Compared to GIRF, MIRF does not depend on an extreme identification. The estimation accuracy is therefore improved, and the associated FEVD is no longer unexplainable. To illustrate this, an empirical example based on West German macroeconomic series is further provided. Hence, the proposed MIRF can be a potentially useful tool to study interdependence of multivariate time series in finance practices.

2 | THE VECTOR AUTOREGRESSIVE MODEL AND EXISTING APPROACHES OF THE IMPULSE RESPONSE FUNCTION

Define $\mathbf{y}_t = (y_{1,t}, \dots, y_{N,t})'$, an $N \times 1$ vector of time series variables observed at time t for $t = 1, \dots, T$, a p-th order vector autoregressive model, denoted by VAR(p), can be written as:

$$\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2\mathbf{y}_{t-2} + \dots + \mathbf{A}_p\mathbf{y}_{t-p} + \epsilon_t, \quad t = 1, \dots, T, \tag{1}$$

where $\boldsymbol{\mu}$ is an N-dimensional intercept vector, $\mathbf{A}_i$ for $i = 1, \dots, p$ are $N \times N$ coefficient matrices that evaluate the long-run co-movement between time series variables at time t and those at time $t-i$, and ϵ_t is an $N \times 1$ zero-mean white noise vector process (serially independent) with a variance–covariance matrix $\boldsymbol{\Sigma}_\epsilon$.

Based on a VAR system, one can compute the impulse response function (IRF), an important metric to measure the responses of $\mathbf{y}_t$ to interested shocks. When covariance stationary, (1) can be written as a moving-average (MA) model, such that

$$\mathbf{y}_t = \mathbf{c} + \boldsymbol{\Phi}_1\epsilon_{t-1} + \boldsymbol{\Phi}_2\epsilon_{t-2} + \dots \tag{2}$$

However, due to the usual non-zero cross-sectional correlations among the error sequence, elements of $\boldsymbol{\Phi}_s$ for $s > 0$ cannot be easily interpreted. To resolve that, Sims (1980) recommends working on the orthogonal errors, or $\boldsymbol{\eta}_t = \mathbf{P}^{-1}\epsilon_t$, where $\boldsymbol{P}$ is an invertible lower triangular matrix derived using the Cholesky decomposition of $\boldsymbol{\Sigma}_\epsilon$. The MA representation (2) can then be rewritten with respect to $\boldsymbol{\eta}_t$:

$$\mathbf{y}_t = \mathbf{c} + \boldsymbol{\Theta}_1\boldsymbol{\eta}_{t-1} + \boldsymbol{\Theta}_2\boldsymbol{\eta}_{t-2} + \dots$$

IRFs are therefore defined as impulse responses of the i-th series to the j-th orthogonal shock $\eta_{j,t}$, or

$$\frac{\partial y_{i,t+s}}{\partial \eta_{j,t}} = \frac{\partial y_{i,t}}{\partial \eta_{j,t-s}} = \theta^o_{ij,s}, i,j = 1,...,N; s > 0$$

where $\theta^o_{ij,s}$ is the *ij*-th element of $\mathbf{\Theta}_s$, defined as the orthogonal IRF (OIRF).

The measure forecast error variance decomposition (FEVD) is often discussed with IRF. In short, the *ij*-th *h*-step-ahead FEVD describes the portion of variance of forecast errors in predicting $y_{i,T+h}$ explained by the *j*-th orthogonal shocks $\eta_{j,t}$. The mathematical definition is provided below.

$$\omega^o_{ij,h} = \sum_{s=0}^{h-1} \left(\mathbf{e}_i' \mathbf{\Theta}_s \mathbf{e}_j\right)^2 / \text{MSE}_i(h)$$

where $\mathbf{e}_i$ is the *i*-th column of the $N \times N$ identity matrix $\mathbf{I}_N$, and $\text{MSE}_i(h) = \sum_{s=0}^{h-1} \mathbf{e}'_i \mathbf{\Phi}_s \mathbf{\Sigma}_\epsilon \mathbf{\Phi}_s' \mathbf{e}_i$. $\omega^o_{ij,h}$ is defined as the orthogonal FEVD (OFEVD).

> *Remark* 1. Due to the application of Cholesky decomposition, OIRF and the associated OFEVD are affected by ordering of elements in $\mathbf{y}_t$ and thus are not unique. In practice, however, this ordering is essential to the estimation of OIRF, especially for those measuring the contemporaneous (i.e., $s = 0$) responses (see section 2.3.4 of Lütkepohl (2007) for details). Without loss of generality, a possible solution is to fit all the $N!$ possible permutations of VAR, and OIRF and OFEVD can be computed as the corresponding averages. However, the computational cost of such an approach will increase exponentially, leading to infeasibility even for an intermediate N.

To resolve this non-uniqueness of OIRF, Koop et al. (1996) define a generalized IRF (GIRF) as $E\left(\mathbf{y}_{t+s} | \epsilon_{jt} = \delta_j, \mathbf{\Omega}_{t-1}\right) - E\left(\mathbf{y}_{t+s} | \mathbf{\Omega}_{t-1}\right)$, where δ_j is a known size of shock and $\mathbf{\Omega}_{t-1}$ is the information set at time $t-1$. As concluded by Pesaran and Shin (1998), GIRF does not require orthogonal errors and is invariant to ordering of series with the following specification:

$$\theta^g_{ij,s} = (\sigma_\epsilon)_{jj}^{-1/2} \mathbf{e}_i' \mathbf{\Phi}_s \mathbf{\Sigma}_\epsilon \mathbf{e}_j,$$

where $(\sigma_\epsilon)^2_{jj}$ is the *j*-th diagonal element of $\mathbf{\Sigma}_\epsilon$. The corresponding generalized FEVD (GFEVD) is computed as

$$\omega^g_{ij,h} = (\sigma_\epsilon)_{jj}^{-1} \sum_{s=0}^{h-1} \left(\mathbf{e}_i' \mathbf{\Phi}_s \mathbf{\Sigma}_\epsilon \mathbf{e}_j\right)^2 / \text{MSE}_i(h).$$

> *Remark* 2. Despite its invariance to ordering of series, GIRF has some outstanding drawbacks. Unlike OFEVD, or $\omega^o_{ij,h}$, which always sums up to 1 for each response *j*, the GFEVD counterpart $\omega^g_{ij,h}$ will not add to 1 in general. This is caused by the off-diagonal non-zeros of $\mathbf{\Sigma}_\epsilon$. Consequently, an interpretation would be difficult for GFEVD, as the total explained proportions of variation exceed 100%. More importantly, as $\phi^o_{ij,h} = \phi^g_{ij,h}$ only applies for $j = 1$ (Pesaran & Shin, 1998), it is straightforward to infer that $\phi^g_{ij,h}$ is equivalent to the OIRF counterpart with the *j*-th variable ranked first in a new VAR system. Thus, this may be deemed not general in effect but a rather extreme identifying assumption (Kim, 2013).

3 | A MODIFIED IMPULSE RESPONSE FUNCTION

The source of variance to ordering of series for OIRF is the employed Cholesky decomposition. Thus, a natural alternative approach would be the eigendecomposition. Motivated by this, we propose a modified IRF (MIRF) as follows

$$\theta^{m}_{ij,s} = \mathbf{e}_i'\mathbf{\Phi}_s\mathbf{F}\mathbf{e}_j \text{ and } \mathbf{F} = \left(\mathbf{G}\mathbf{\Lambda}^{-1/2}\mathbf{G}'\right)^{-1}, \tag{3}$$

where $\mathbf{\Lambda}$ is an $N \times N$ diagonal matrix containing the eigenvalues of $\mathbf{\Sigma}_\epsilon$, ranked from largest to smallest, and $\boldsymbol{G}$ is the corresponding eigenvectors. Under fairly general assumptions for a covariance stationary VAR system, it is straightforward to see that $\boldsymbol{F}$ is a positive definite matrix.

Remark 3. Following the property of eigendecomposition, one can show that $\mathbf{F} = \mathbf{G}\mathbf{\Lambda}^{1/2}\mathbf{G}'$ and therefore $\mathbf{\Sigma}_\epsilon = \mathbf{F}\mathbf{F}$. Let $\boldsymbol{\zeta}_t = \mathbf{F}^{-1}\epsilon_t$, we have that

$$E(\boldsymbol{\zeta}_t\boldsymbol{\zeta}'_t) = E\left(\mathbf{F}^{-1}\epsilon_t\epsilon'_t\mathbf{F}^{-1}\right) = \mathbf{F}^{-1}\mathbf{\Sigma}_\epsilon\mathbf{F}^{-1} = \mathbf{I}_N.$$

Thus, the implied decomposition to compute MIRF provides another set of orthogonal errors in (1).

The associated modified FEVD of (3), or MFEVD, is defined below

$$\omega^{m}_{ij,h} = \sum_{s=0}^{h-1} \left(\mathbf{e}_i'\mathbf{\Phi}_s\mathbf{F}\mathbf{e}_j\right)^2 / \mathrm{MSE}_i(h). \tag{4}$$

Hence, it is easy to show that $\sum_{j=1}^{N}\omega^{m}_{ij,h} = 1$ for responses of the i-th variable. This makes the relevant interpretation straightforward and comparable to that of OIRF. We now prove the uniqueness of MIRF and MFEVD.

Remark 4. The rationale of MIRF can be illustrated in two steps and is closely related to the principal component analysis. First, the i-th principal component (PC) of ϵ_t is the i-th element of $\mathbf{G}'\epsilon_t$. After scaling to a unit variance, those orthogonal PCs can be denoted by $\mathbf{p}_t = \mathbf{\Lambda}^{-1/2}\mathbf{G}'\epsilon_t$. Second, orthogonal idiosyncratic errors are then produced using the PCs. Specifically, note that proportional contributions of the i-th variable to those PCs (i.e., loadings) are essentially the i-th row of $\boldsymbol{G}$. Thus, the i-th element of $\boldsymbol{\eta}^E_t = \mathbf{G}\mathbf{p}_t = \mathbf{G}\left(\mathbf{\Lambda}^{-1/2}\mathbf{G}'\epsilon_t\right) = \mathbf{F}^{-1}\epsilon_t$ is the summation of contributions of the i-th variable in all PCs. $\eta^E_{i,t}$ then naturally constitutes the orthogonal error of the i-th variable, which is obtained via the eigendecomposition. The interpretation of MIRF is therefore much similar to that of OIRF, except that the orthogonal errors are produced via the eigendecomposition, such that no recursive casual relationship is needed or assumed.

Proposition 1. *The MIRF and MFEVD defined in (3) and (4), respectively, are independent of the ordering of series.*

Proof. Let $\mathbf{E}_{ij}$ denote the elementary matrix obtained by interchanging the i-th and the j-th rows of an $N \times N$ identity matrix. Thus, $\mathbf{E}_{ij} = \mathbf{E}_{ij}^T$ and $\mathbf{E}_{ij}^{-1} = \mathbf{E}_{ij}$. Also, if $\mathbf{g}_l$ is an eigenvector of a square matrix $\boldsymbol{A}$ with the corresponding eigenvalue λ_l, then $\mathbf{E}_{ij}\mathbf{g}_l$ is the eigenvector of $\mathbf{E}_{ij}\mathbf{A}$ corresponding to the same eigenvalue λ_l.

When the i-th and j-th elements in $\mathbf{y}_t$ are interchanged, the resulting new covariance matrix of errors is denoted by $\widetilde{\mathbf{\Sigma}}_\epsilon$, and $\widetilde{\mathbf{\Sigma}}_\epsilon = \mathbf{E}_{ij}\mathbf{\Sigma}_\epsilon\mathbf{E}_{ij}$. However, the order of eigenvalues remain unchanged, and thus $\widetilde{\mathbf{\Lambda}} = \mathbf{\Lambda}$. Further using the properties stated above, one can infer that $\widetilde{\mathbf{G}} = \mathbf{E}_{ij}\mathbf{G}$.

Therefore the interchanged factorization matrix is

$$\widetilde{\mathbf{F}} = \left(\widetilde{\mathbf{G}}\widetilde{\boldsymbol{\Lambda}}^{-1/2}\widetilde{\mathbf{G}}'\right)^{-1} = \widetilde{\mathbf{G}}\widetilde{\boldsymbol{\Lambda}}^{1/2}\widetilde{\mathbf{G}}' = \mathbf{E}_{ij}\mathbf{G}\boldsymbol{\Lambda}^{1/2}\mathbf{G}\mathbf{E}_{ij} = \mathbf{E}_{ij}\mathbf{F}\mathbf{E}_{ij}$$

Further with $\widetilde{\boldsymbol{\Phi}}_h = \mathbf{E}_{ij}\boldsymbol{\Phi}_h\mathbf{E}_{ij}$, it can be shown that

$$\widetilde{\boldsymbol{\Theta}}_h^m = \widetilde{\boldsymbol{\Phi}}_h\widetilde{\mathbf{F}} = \mathbf{E}_{ij}\boldsymbol{\Phi}_s\mathbf{E}_{ij}\mathbf{E}_{ij}\mathbf{F}\mathbf{E}_{ij} = \mathbf{E}_{ij}\boldsymbol{\Theta}_h^m\mathbf{E}_{ij},$$

where $\boldsymbol{\Theta}_h^m$ is the $N \times N$ matrix consisting of $\theta_{ij,h}^m$, and $\widetilde{\boldsymbol{\Theta}}_h^m$ is the interchanged counterpart. Therefore, it is clear that the MIRF is independent of the reordering. The same logic will straightforwardly apply to MFEVD, which completes the proof. □

Based on the asymptotic properties of the maximum likelihood estimator (MLE) of $\boldsymbol{\Phi}$ and $\boldsymbol{\Sigma}_\epsilon$, Lütkepohl (1990) and Pesaran and Shin (1998) have developed asymptotic distributions of OIRF and GIRF, respectively. Motivated by those works, the asymptotic distribution of MIRF is investigated below.

Lemma 1. *Assume that $\boldsymbol{\Sigma}_\epsilon$ is positive definite, and $\mathbf{F}$ is defined in (3), then*

$$\frac{\partial \text{vech}(\mathbf{F})}{\partial \text{vech}(\boldsymbol{\Sigma}_\epsilon)'} = \mathbf{L}(\mathbf{F}\otimes\mathbf{I}_N + \mathbf{I}_N\otimes\mathbf{F})^{-1}\mathbf{D},$$

where $\mathbf{L}$ and $\mathbf{D}$ are the $[0.5\ N \times (N + 1)] \times N^2$ elimination matrix and $N^2 \times [0.5\ N \times (N + 1)]$ duplication matrix, respectively.

Proof. Since $\boldsymbol{\Sigma}_\epsilon = \mathbf{F}\mathbf{F}$, we have that $\text{d}\boldsymbol{\Sigma}_\epsilon = (\text{d}\mathbf{F})\mathbf{F} + \mathbf{F}(\text{d}\mathbf{F})$. Applying matrix differentiation, it can be shown that

$$\text{dvec}(\boldsymbol{\Sigma}_\epsilon) = (\mathbf{F}\otimes\mathbf{I}_N + \mathbf{I}_N\otimes\mathbf{F})\text{dvec}(\mathbf{F})$$

As defined in (3), together with the assumption that $\boldsymbol{\Sigma}_\epsilon$ is positive definite, it is easily shown that $\mathbf{F}$ is also positive definite. Thus, $(\mathbf{F}\otimes\mathbf{I}_N + \mathbf{I}_N\otimes\mathbf{F})$ is invertible and

$$(\mathbf{F}\otimes\mathbf{I}_N + \mathbf{I}_N\otimes\mathbf{F})^{-1}\text{dvec}(\boldsymbol{\Sigma}_\epsilon) = \text{dvec}(\mathbf{F}).$$

Left multiplying $\boldsymbol{L}$ and substituting $\text{dvec}(\boldsymbol{\Sigma}_\epsilon) = \mathbf{D}\text{dvech}(\boldsymbol{\Sigma}_\epsilon)$, we have that

$$\frac{\partial \text{vech}(\mathbf{F})}{\partial \text{vech}(\boldsymbol{\Sigma}_\epsilon)'} = \mathbf{L}(\mathbf{F}\otimes\mathbf{I}_N + \mathbf{I}_N\otimes\mathbf{F})^{-1}\mathbf{D},$$

which completes the proof.

Using Lemma 1, the following proposition can be straightforwardly proved, together with the proof of proposition 3.6 in Lütkepohl (2007), as $\partial\text{vec}(\mathbf{F})/\partial\text{vech}(\boldsymbol{\Sigma}_\epsilon)' = \mathbf{D}\partial\text{vech}(\mathbf{F})/\partial\text{vech}(\boldsymbol{\Sigma}_\epsilon)'$.

Proposition 2. *Suppose that the MLE of coefficients $\mathbf{A}_s$ ($s = 1,..., p$) and $\boldsymbol{\Sigma}_\epsilon$ follow assumptions of proposition 3.6 in* Lütkepohl (2007)*. Then, the MLE of MIRF follows that*

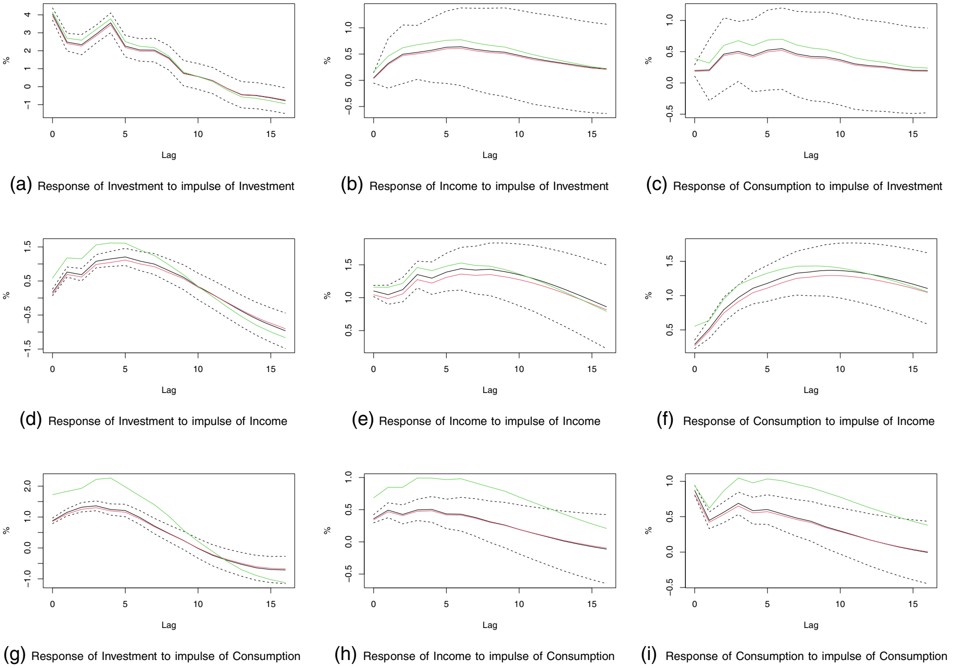
$$\sqrt{T}\text{vec}\left(\widehat{\boldsymbol{\Theta}}_h^m - \boldsymbol{\Theta}_h^m\right) \xrightarrow{d} \mathcal{N}\left(0, \mathbf{C}_h\boldsymbol{\Sigma}_\alpha\mathbf{C}_h' + \overline{\mathbf{C}}_h\boldsymbol{\Sigma}_\alpha\overline{\mathbf{C}}_h'\right), \quad h = 0, 1, 2, \ldots$$

where $\rightarrow d$ *indicates the asymptotic convergence in distribution.* $\mathbf{C}_0 = \mathbf{0}$, *and* $\mathbf{C}_h = (\mathbf{F} \otimes \mathbf{I}_N)\mathbf{G}_h$ *for* $h > 0$. $\overline{\mathbf{C}}_h = (\mathbf{I}_N \otimes \mathbf{\Phi}_h)\mathbf{H}$, *and* $\mathbf{H} = \partial \text{vec}(\mathbf{F}) / \partial \text{vech}(\mathbf{\Sigma}_\epsilon)' = \mathbf{DL}(\mathbf{F} \otimes \mathbf{I}_N + \mathbf{I}_N \otimes \mathbf{F})^{-1}\mathbf{D}$. $\mathbf{\Sigma}_\alpha$ *and* $\mathbf{G}_h$ *for* $h > 0$ *are defined as in Proposition 3.6 of* Lütkepohl (2007).

Therefore, the proposed MIRF retains all desirable properties of OIRF, such as the orthogonal decomposition, explainable associated FEVD and well-defined asymptotic properties. Further with the proved uniqueness, MIRF could be a widely useful alternative to study the impulse responses, especially for a large VAR system.

4 | AN ILLUSTRATIVE EMPIRICAL EXAMPLE

For illustration purpose, we re-investigate the empirical data examined in Lütkepohl (2007). The dataset is a three-dimensional system consisting of the logarithms of quarterly, seasonally adjusted West German fixed investment (Investment), disposable income (Income), and consumption expenditures (Consumption) over 1962–1982. Using the Bayesian information criterion, we choose a VAR with order 5. The largest eigenvalue of the companion coefficient matrix (see page 118 of Lütkepohl (1990) for the matrix denoted by ***A*** therein) is 0.979, suggesting the covariance stationarity of the fitted VAR system.



(a) Response of Investment to impulse of Investment (b) Response of Income to impulse of Investment (c) Response of Consumption to impulse of Investment

(d) Response of Investment to impulse of Income (e) Response of Income to impulse of Income (f) Response of Consumption to impulse of Income

(g) Response of Investment to impulse of Consumption (h) Response of Income to impulse of Consumption (i) Response of Consumption to impulse of Consumption

FIGURE 1 Fitted impulse response functions. (a) Response of investment to impulse of investment. (b) Response of income to impulse of investment. (c) Response of consumption to impulse of investment. (d) Response of investment to impulse of income. (e) Response of income to impulse of income. (f) Response of consumption to impulse of income. (g) Response of investment to impulse of consumption. (h) Response of income to impulse of consumption. (i) Response of consumption to impulse of consumption. This figure plots the fitted impulse response functions of the investigated West German macroeconomic series, spanning over 1962–1982. Black, red and green curves are the point estimates of the MIRF, OIRF and GIRF. Dashed lines are the 68% confidence intervals of the MIRF

TABLE 1 Estimated FEVD

	OFEVD			MFEVD			GFEVD		
Step	Investment	Income	Consumption	Investment	Income	Consumption	Investment	Income	Consumption
Panel A: Responses of investment									
1	90.5%	0.9%	8.6%	95.3%	0.2%	4.5%	100.0%	2.0%	17.3%
4	77.9%	5.8%	16.3%	82.2%	5.0%	12.8%	92.9%	12.4%	33.6%
8	74.2%	9.1%	16.7%	78.3%	9.0%	12.7%	89.7%	17.8%	36.0%
12	73.7%	9.8%	16.4%	77.8%	9.8%	12.4%	89.0%	18.6%	35.5%
16	71.9%	10.7%	17.4%	75.8%	10.7%	13.4%	87.2%	20.1%	37.0%
Panel B: Responses of income									
1	0.9%	81.9%	17.3%	0.2%	90.3%	9.6%	2.0%	100.0%	34.8%
4	9.2%	71.5%	19.3%	9.3%	79.0%	11.7%	15.8%	92.3%	42.3%
8	12.1%	70.4%	17.5%	12.6%	78.0%	9.4%	19.6%	89.5%	39.3%
12	11.6%	72.7%	15.7%	12.2%	80.8%	7.0%	18.3%	89.3%	34.6%
16	10.9%	74.4%	14.7%	11.4%	82.7%	5.9%	16.8%	88.6%	30.9%
Panel C: Responses of consumption									
1	7.6%	16.4%	76.0%	4.5%	9.6%	85.9%	17.3%	34.8%	100.0%
4	13.7%	44.2%	42.1%	13.0%	45.6%	41.4%	25.4%	69.7%	73.6%
8	12.4%	58.8%	28.8%	12.4%	63.6%	24.0%	22.3%	82.5%	56.9%
12	10.1%	66.8%	23.0%	10.2%	73.3%	16.5%	18.0%	87.2%	46.6%
16	8.6%	71.3%	20.1%	8.6%	78.6%	12.8%	15.1%	88.4%	39.7%

Note: This table presents the fitted forecast error variance decomposition of the investigated West German macroeconomic series, spanning over 1962–1982.

We now compare IRF and FEVD computed using the three approaches. To compute OIRF, as described in Section 2, we fit all the six (3!) permutations of VAR(5). The OIRF and OFEVD are then calculated as the corresponding averages of the six estimates. Estimated IRFs over periods 0–16 (4 years) are plotted in Figure 1. In addition to the point estimates of OIRF, MIRF, and GIRF, we also produce 68% confidence intervals of MIRF, by adding/subtracting one asymptotic standard error defined in Proposition 2. In their seminal work, Sims and Zha (1999) support the examination of such one-standard-error intervals over the popular 95% or 99% counterparts, as use of high-probability intervals camouflages the occurrence of large errors of over-coverage. In the vast majority of cases, we observe that point estimates of OIRF and MIRF are fairly close to each other. GIRF can considerably deviate from them, with potentially significant differences in many cases. Consistent with Kim (2013), we find that using GIRF could result in more extreme are potentially 'wrong' inferences. For instance, the influences of shocks of Income and Consumption may be deemed too high, especially over short terms. Finally, as displayed in Table 1, OFEVD and MFEVD add up to 100% for each response. The summation of GFEVD exceeds 100% in all steps for all responses, leading to difficulty in explanation. Even when those GFEVD are (groundlessly) rescaled to 100%, the estimates are substantially different from OFEVD and MFEVD, which are relatively closer.

5 | CONCLUDING REMARKS

The proposed MIRF retains all attractive properties of OIRF and successfully resolves its issue of non-uniqueness. Relative to early attempts of unique measures, such as GIRF, the superiority of MIRF includes the non-extreme identification and explainable associated FEVD. Compared to unique measures based on more recent developments, such as that investigated in Lanne and Luoto (2016), MIRF only requires the general VAR model, and thus the estimation is computationally efficient. As for unique OIRF computed over averages of all VAR permutations, MIRF can reduce the computational insensitivity to a minimum level. Those merits of MIRF make it an appealing approach to study responses and error decompositions to idiosyncratic shocks in finance practices, especially when prior knowledge to identify a justifiable ordering (i.e., recursive casual relationship) of variables is not available.

As evidenced in Section 4, estimates of MIRF are much closer to those of OIRF based on averages. This implies the desirable accuracy of MIRF over other competitors such as GIRF. Future studies may more systematically explore the estimation accuracy of MIRF.

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CONFLICT OF INTEREST

The author declares that there is no conflict of interest.

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