DEMAND SYSTEMS INCORPORATING INTERTEMPORAL CONSUMPTION DYNAMICS

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Abstract

This paper integrates two strands of studies on consumer demand and consumption and evaluates the relevance of the traditional approach for analyzing consumer behavior employing an intertemporal two-stage budgeting procedure. We take a modified AIDS framework for the demand system and derive a general Euler equation by allowing for life-cycle behavior and precautionary saving. We also investigate excess sensitivity and habit formation in consumption behavior. The demand system and the Euler equation constitute a system of recursive equations with cross-equation parameter restrictions. Joint estimation of the demand system and the Euler equation can lead to substantial efficiency gains. Separate or sequential estimation tends to bias parameter estimates and the degree of risk aversion and intertemporal substitution.

Key Words: Indirect utility function, MAIDS, Two-stage budgeting, Euler equation, Precautionary Saving

JEL Numbers: D12, D91, E21

I. INTRODUCTION

Consumption constitutes two-thirds of most countries' output measured by GDP (Gross Domestic Product) and is inextricably related to the level of saving that provides capital for productive investment and future consumption. Thus fluctuations in consumption have a great impact on the economy, and a proper understanding of consumer behavior is important. There are many theories of consumer behavior, but traditionally studies are conducted at two separate levels in microeconomics and macroeconomics. Micro analysis of consumer behavior is concerned with optimal allocation of the consumer's given expenditure to different goods by estimating demand systems conditional on the level of total expenditure (see Deaton and Meullbauer, (1980) for an extensive discussion). In macro analysis, in contrast, research on consumer behavior involves allocation of the consumer's wealth across periods by estimating the life-cycle or intertemporal pattern of consumption (see Deaton (1992) for an excellent survey).

While the separation of research into consumer demand and consumption has proved to be useful, it fails to address many important issues facing consumers. Studies on consumer demand are a static analysis and do not contribute to an understanding of the intertemporal behavior of consumers. Consumption studies, on the other hand, are highly aggregated and fail to provide disaggregated information about its components. In particular, the definition of consumption is left vague, to be addressed at the empirical stage. To rectify the problems arising from separate research in consumer behavior, there have been only a few recent attempts at integrating consumer demand and intertemporal consumption functions. Blundell, Browning, and Meghir(1994) and Attanasio and Weber (1995) estimate the AIDS demand system and then use the parameters from this demand system to estimate the Euler equation using synthetic panel data. Their models allow for non-homothetic within-period preferences and thus account for changes in relative prices in intertemporal consumption with the effect of producing a high intertemporal elasticity of substitution in consumption, in sharp contrast to earlier studies based on homothetic preferences with highly aggregate data (see Hall, 1988). Their procedure, however, may be inefficient because the demand system and the Euler equation are estimated sequentially without allowing for cross-equation parameter restrictions. Instead, while the demand system and the Euler equation constitute a system of recursive equations, the existence of cross-equation parameter restrictions requires that the system needs to be jointly estimated.

This study further integrates two strands of studies on consumer demand and consumption in a way that allows a number of interesting questions to be addressed. In particular, what are the effects on the parameter estimates of the demand system if it is estimated jointly with, instead of in isolation from, the Euler equation? Does joint estimation of the demand system and the Euler equation provide more insights into the continuing debate on the appropriate estimation of the elasticity of intertemporal substitution? Is there an efficiency gain from joint estimation relative to the sequential estimation adopted by Blundell, Browning, and Meghir(1994) and Attanasio and Weber (1995)? Moreover, studies on consumer demand typically employ Marshallian elasticities as relevant price elasticities of demand. However, to the extent that these elasticities are derived under the condition that total expenditure is given, they do not represent true elasticities in an intertemporal environment. A natural question is: what are the appropriate elasticities when we allow for consumption expenditure to be endogenously determined by jointly estimating consumer demand and the Euler equation?

To address these issues, we propose a general framework for analyzing consumer behavior by integrating the demand system and the Euler equation. We take a modified AIDS framework for the demand system and derive a general Euler equation by allowing for life-cycle behavior and precautionary saving. The proposed model is more general than the ones employed in earlier studies and allows intertemporal substitution and risk aversion to be disentangled. In existing studies, the same parameter controls risk aversion and intertemporal substitution, and the intertemporal elasticity of substitution is taken to be the reciprocal of the degree of relative risk aversion. We also investigate excess sensitivity of consumption to income and habit formation in consumption behavior. In estimation, we present a new procedure to allow for autocorrelation in share equations which is compatible with the model and use an ARCH-type procedure to estimate a precautionary saving measure. Results show that joint estimation of the demand system and the Euler equation can lead to substantial efficiency gains.

II AN INTEGRATED MODEL OF CONSUMER DEMAND AND INTERTEMPORAL CONSUMPTION

A. Theoretical Framework

We take the intertmeporal two-stage budgeting procedure as a point of departure. In this procedure, the consumer allocates intertemporal wealth across periods at the first stage, while each period's optimal allocation of wealth is distributed across commodities at the second stage. The first-stage optimization problem gives the consumption function, and the second-stage optimization problem yields the demand systems. An explicit solution for the first-stage intertemporal optimization problem is often difficult because it depends on complex variables involving wealth and expectation variables, and it is a common practice to work with the Euler equation to circumvent this problem (see Hall, 1978). While the two-state budgeting procedure allows the consumer's intratemporal and intertemporal decisions can be separated, it also provides a consistent framework to integrate the demand system and the Euler equation.

In the terminology of Blackborby, Primont, and Russell (1978), the second stage is related to decentralizability, while the first stage corresponds to price aggregation. Decentralizability is satisfied by the usual assumption of intertemporal additivity of preferences, and the second stage optimization requires knowledge only of the within-period level of consumption and within-period prices of goods. The first stage optimization of solving for consumption can then be specified conditional on the optimal allocation of this level of consumption in each period. The existence of the price indices required for this first stage optimization then follow naturally from the specification of the price indices that occur in the indirect utility function.

The consumer's second stage optimization problem is summarized by the indirect utility function $V(C_t, \mathbf{p}_t)$ defined by

$$V(C_t,\, \mathbf{p}_t) = \max_{\mathbf{q}_t}\{u(\mathbf{q}_t) \mid \mathbf{p}_t \cdot \mathbf{q}_t \le C_t\}, \tag{1}$$

where C_t is total expenditure to be allocated among different commodities at period t (t = 0, 1, …, ∞) and $\mathbf{p}_t$ is a vector of commodity prices whose elements are p_{it} (i = 1, .., n), the price of the ith commodity at period t, and $\mathbf{q}_t$ is a vector of commodities whose elements are q_{it} (i = 1, …, n), the quantity of the ith commodity at period t,. The indirect utility function (1) possesses usual regularity conditions. Applying Roy's identity, we obtain the commodity demand functions:

$$q_{it}(C_t,\mathbf{p}_t)=\frac{\partial V(C_t,\mathbf{p}_t)/\partial p_{it}}{\partial V(C_t,\mathbf{p}_t)/\partial C_t}.\ (i=1,...,n) \tag{2}$$

While the intratemporal allocation of consumption across goods as captured by (2) is invariant to a monotonic transformation of a utility function (1), intertemporal allocation of consumption is not. Thus for the first-stage optimization problem of the intertemporal two-stage budgeting procedure, we take the following lifetime or intertemporal utility function:

$$U=\sum_{t=0}^{\infty}\frac{1}{(1+\rho)^t}\frac{1}{1-\zeta}\{V(C_t,\mathbf{p}_t)\}^{1-\zeta}, \tag{3}$$

where ρ is the constant rate of the consumer's time preference and ζ is a parameter that produces a transformation of the indirect utility function. Thus time-separable transformed within-period preferences are a power function of the indirect utility function. As ζ approaches one, $1/(1-\zeta)\{V(C_t,\mathbf{p}_t)\}^{1-\zeta}=\ln V(C_t,\mathbf{p}_t)$. When $\zeta = 0$, the intertemporal utility function is simply the discounted sum of within-period utilities $V(C_t,\mathbf{p}_t)$. Note that if the indirect utility function is homothetic that allows us to define a price indix $P_t(\mathbf{p}_t)$ at each period and if the indirect utility function is written as $V(C_t,\mathbf{p}_t)=c_t\equiv C_t/P_t$, where c_t is real consumption, then we have the familiar, conventional lifetime utility function of the form:

$$U=\sum_{t=0}^{\infty}\frac{1}{(1+\rho)^t}\frac{1}{1-\zeta}c_t^{1-\zeta}. \tag{4}$$

Given the transformation of the indirect utility function, the marginal utility of consumption at time t (λ_t) is obtained as

$$\lambda_t\equiv\frac{\partial U}{\partial C_t}=\frac{V(C_t,\mathbf{p}_t)^{-\zeta}}{(1+\rho)^t}\frac{\partial V(C_t,\mathbf{p}_t)}{\partial C_t}. \tag{5}$$

so that

$$\frac{\partial \lambda_t}{\partial C_t} = \frac{1}{(1+\rho)^t}\left[V(C_t,\mathbf{p}_t)^{-\zeta}\frac{\partial^2 V(C_t,\mathbf{p}_t)}{\partial C_t^2} - \zeta V(C_t,\mathbf{p}_t)^{-(\zeta+1)}\left(\frac{\partial V(C_t,\mathbf{p}_t)}{\partial C_t}\right)^2\right]. \quad (6)$$

Intertemporal optimization requires that the intertermporal utility function (3) is concave, which implies that $\partial \lambda_t / \partial C_t < 0$. For $\zeta > 0$, the concavity of the indirect utility function ($\partial^2 V(C_t,\mathbf{p}_t)/\partial C_t^2$) guarantees the concavity of the intertemporal utility function. In this case, if wealth increases, λ_t decreases and, given this decline in λ_t, consumption increases. However, if $\zeta < 0$, even if the indirect utility function is concave, the intertemporal utility function can be convex so that $\partial \lambda_t / \partial C_t > 0$. Thus it is clear that the parameter ζ is essential in determining the marginal utility of consumption and its curvature. Note, however, that this parameter cannot be identified with the demand system; it is identified with the intertemporal optimization decision.

Maximization of lifetime utility (3) under uncertainty subject to the consumer's intertemporal budget constraint with perfect capital markets yields the well-known Euler equation (see Deaton, 1992):

$$E_t[(1+r_{t+1})\lambda_{t+1}/\lambda_t] = 1\ , \quad (7)$$

where E_t is an expectation operator conditional on information available at time t and r_{t+1} is the nominal rate of interest earned on assets held between period t and t+1. A closed-form solution for the Euler equation (7), however, is not in general possible. We assume that consumers have rational expectations and exploit a log approximation property. Under rational expectations, we can write (7) as

$$(1+r_{t+1})\lambda_{t+1}/\lambda_t = 1 + \varepsilon_{t+1}\ , \quad (8)$$

where ε_{t+1} is an expectation error at time t+1 that is uncorrelated with variables known at time t such that $E_t(\varepsilon_{t+1}) = 0$ and $Var_t(\varepsilon_{t+1}) = \sigma_{t+1}^2$. Now take logs on both sides of (8) and use the second-order approximation for $\ln(1+\varepsilon_{t+1})$ around ε_{t+1} = 0, i.e., $\ln(1+\varepsilon_{t+1}) = \varepsilon_{t+1}$ − i.e., $(1/2)\varepsilon_{t+1}^2$. The approximation holds exactly if $(1+\varepsilon_{t+1})$ is lognormally distributed. The result gives us

$$\ln(\lambda_{t+1}/\lambda_t) = -\ln(1+r_{t+1}) - (1/2)\sigma_{t+1}^2 + u_{t+1}, \tag{9}$$

where $u_{t+1} = \varepsilon_{t+1} + (1/2)(\sigma_{t+1}^2 - \varepsilon_{t+1}^2)$ with $E_t(u_{t+1}) = 0$. When the rate of return is risk free, σ_{t+1}^2 is proportional to the conditional variance of consumption growth innovation, which captures the precautionary saving motive. Equation (9) reveals the life-cycle and precautionary saving motive of consumption behavior such that the growth of marginal utility is determined by the interest rate, precautionary saving, and unexpected shock.

B. Model Structure

This discussion suggests that we need to specify the indirect utility function (1) for empirical analysis. We take a generalization of the indirect utility function of the extended MAIDS system proposed by Cooper and McLaren (1992). The MAIDS produces a fractional demand system in which shares are constrained to lie within the unit interval and can be shown to be more regular than the original AIDS (Deaton and Meullbauer, 1980) in which shares are a linear function of (log) expenditure. For this version of MAIDS, the consumer's within-period preferences (1) are represented by the following indirect utility function:

$$V(C_t, \mathbf{p}_t) = \frac{C_t^{1-\eta}}{G(\mathbf{p}_t)H(\mathbf{p}_t)^{-\eta}} \ln\left(\frac{C_t}{A(\mathbf{p}_t)}\right), \tag{10}$$

where the parameter takes values $\eta \le 1$, and $G(p_t)$, $H(p_t)$, and $A(p_t)$ are concave linear homogeneous price indexes, which are given by

$$\ln G(\mathbf{p}_t) = \sum_{j=1}^{n} \beta_j \ln p_{jt} \text{ with } \sum_{j=1}^{n} \beta_j = 1, \tag{11}$$

$$\ln H(\mathbf{p}_t) = \sum_{j=1}^{n} \alpha_j \ln p_{jt} \text{ with } \sum_{j=1}^{n} \alpha_j = 1, \tag{12}$$

$$\ln A(\mathbf{p}_t) = \sum_{j=1}^{n} \alpha_j \ln p_{jt} + 1/2 \sum_{j=1}^{n} \sum_{k=1}^{n} \gamma_{jk} \ln p_{jt} \ln p_{kt} \text{ with } \sum_{k=1}^{n} \gamma_{jk} = 0 \text{ and } \gamma_{jk} = \gamma_{kj} \; (k \ne j). \tag{13}$$

The indirect utility function (10) will be concave in C_t (as required for the intertemporal problem) provided that $\ln C_t > (1-2\eta)/\eta(1-\eta)$. A sufficient condition for this to hold is that $C_t > 1$ and $0.5 < \eta < 1$. From Roy's identity, we obtain the share equations of the form:

$$S_{it} = \frac{\alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_{jt} + (\beta_i - \eta\alpha_i)\ln(C_t / A(\mathbf{p}_t))}{1 + (1-\eta)\ln(C_t / A(\mathbf{p}_t))}, \; i = 1, ..., n, \tag{14}$$

where S_{it} is expenditure share of the *i*th (i = 1, .., n) commodity at period t, whose sum adds to 1 ($\sum_{i=1}^{n} S_{it} = 1$).

The AIDS demand system ((Deaton and Meullbauer, 1980) has been widely used in empirical studies for its simplicity and flexibility. The MAIDS is an integrable rank two demand system and contains AIDS as a nested model when $\eta = 1$. In particular, the AIDS indirect utility function is

$$V(C_t, \mathbf{p}_t) = \frac{1}{B(\mathbf{p}_t)} \ln\left(\frac{C_t}{A(\mathbf{p}_t)}\right), \tag{15}$$

where $\ln B(\mathbf{p}_t) = \ln[G(\mathbf{p}_t)/H(\mathbf{p}_t)] = \sum_{j=1}^{n} \tilde{\beta}_j \ln p_{jt}, \tilde{\beta}_j = \beta_j - \alpha_j$, and $\sum_{j=1}^{n} \tilde{\beta}_j = 0$. From this utility function, we derive the familiar AIDS share equations:

$$S_{it} = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_{jt} + \tilde{\beta}_i \ln(C_t / A(\mathbf{p}_t)), \; i = 1, ..., n. \tag{16}$$

The MAIDS system is flexible enough to allow for non-homothetic preferences. When $\tilde{\beta} = 0$ in addition to the restriction that η =1, the MAIDS system produces homothetic preferences.

To operationalize (9) for empirical analysis, we need to derive an expression for λ_{t+1}/λ_t. To this end, substituting for $V(C_t, \mathbf{p}_t)$ in (10), equation (3) is written as

$$U = \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} \frac{1}{1-\zeta} \left\{ \frac{C_t^{1-\eta}}{G(\mathbf{p}_t) H(\mathbf{p}_t)^{-\eta}} \ln\left(\frac{C_t}{A(\mathbf{p}_t)}\right) \right\}^{1-\zeta}, \tag{17}$$

which gives

$$\lambda_t = \frac{1}{(1+\rho)^t} \left[\frac{C_t^{1-\eta}}{G(\mathbf{p}_t) H(\mathbf{p}_t)^{-\eta}} \ln\left(\frac{C_t}{A(\mathbf{p}_t)}\right) \right]^{-\zeta} \frac{C_t^{-\eta}}{G(\mathbf{p}_t) H(\mathbf{p}_t)^{-\eta}} \left[1 + (1-\eta) \ln\left(\frac{C_t}{A(\mathbf{p}_t)}\right) \right]. \tag{18}$$

Hence we can obtain

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{1}{1+\rho}\left[\left(\frac{C_{t+1}}{C_t}\right)^{1-\eta} \frac{G(\mathbf{p}_t)}{G(\mathbf{p}_{t+1})}\left(\frac{H(\mathbf{p}_t)}{H(\mathbf{p}_{t+1})}\right)^{-\eta} \frac{\ln(C_{t+1}/A(\mathbf{p}_{t+1}))}{\ln(C_t/A(\mathbf{p}_t))}\right]^{-\zeta}$$
$$\times\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\eta} \frac{G(\mathbf{p}_t)}{G(\mathbf{p}_{t+1})}\left(\frac{H(\mathbf{p}_t)}{H(\mathbf{p}_{t+1})}\right)^{-\eta}\right]\times\left[\frac{1+(1-\eta)\ln(C_{t+1}/A(\mathbf{p}_{t+1}))}{1+(1-\eta)\ln(C_t/A(\mathbf{p}_t))}\right]. \tag{19}$$

We take logs on both sides of (19) and exploit the approximation property ($\ln(1+x)=x$ for a small value of x) to get

$$\ln\left(\frac{\lambda_{t+1}}{\lambda_t}\right) = -\rho + \left[(1-2\eta-\zeta(2-\eta)\right]\ln\left(\frac{C_{t+1}}{C_t}\right) - (1-\zeta)\ln\left(\frac{G(\mathbf{p}_{t+1})}{G(\mathbf{p}_t)}\right)$$
$$+\ \eta(1-\zeta)\ln\left(\frac{H(\mathbf{p}_{t+1})}{H(\mathbf{p}_t)}\right) - (1-\eta-\zeta)\ln\left(\frac{A(\mathbf{p}_{t+1})}{A(\mathbf{p}_t)}\right). \tag{20}$$

Now use (20) to substitute out ln $(\lambda_{t+1}/\lambda_t)$ in (9) and rearrange to obtain the following expression for consumption growth:

$$\ln\left(\frac{C_{t+1}}{C_t}\right) = \frac{1}{2\eta+\zeta(2-\eta)-1}\left\{\left[r_{t+1} - \ln\left(\frac{A(\mathbf{p}_{t+1})}{A(\mathbf{p}_t)}\right) - \rho\right] + (\eta+\zeta)\ln\left(\frac{A(\mathbf{p}_{t+1})}{A(\mathbf{p}_t)}\right)\right.$$
$$\left. +(\zeta-1)\ln\left(\frac{G(\mathbf{p}_{t+1})}{G(\mathbf{p}_t)}\right) + \eta(1-\zeta)\ln\left(\frac{H(\mathbf{p}_{t+1})}{H(\mathbf{p}_t)}\right) + \frac{1}{2}\sigma_{t+1}^2 - u_{t+1}\right\}, \tag{21}$$

where ($r_{t+1} - \ln(A(\mathbf{p}_{t+1})/A(\mathbf{p}_t))$ is the real interest rate.

For easy interpretation of (21), we can rearrange terms and express it as

$$\ln\left(\frac{C_{t+1}}{C_t}\right)^* \equiv \ln\left(\frac{C_{t+1}}{C_t}\right) - \frac{1}{2\eta+\zeta(2-\eta)-1}\left\{(\eta+\zeta)\ln\left(\frac{A(\mathbf{p}_{t+1})}{A(\mathbf{p}_t)}\right)\right.$$
$$\left. +(\zeta-1)\ln\left(\frac{G(\mathbf{p}_{t+1})}{G(\mathbf{p}_t)}\right) + \eta(1-\zeta)\ln\left(\frac{H(\mathbf{p}_{t+1}}{H(\mathbf{p}_t)}\right)\right\}\Bigg\} \tag{22}$$
$$= \frac{1}{2\eta+\zeta(2-\eta)-1}\left\{\left[(r_{t+1} - \ln\left(\frac{A(\mathbf{p}_{t+1})}{A(\mathbf{p}_t)}\right) - \rho\right] + \frac{1}{2}\sigma_{t+1}^2 - u_{t+1}\right\},$$

where $\ln(C_{t+1}/C_t)^*$ is real consumption growth defined with three price indexes. This equation identifies three sources of predictable real consumption growth – changes in the real interest rate ($r_{t+1} - \ln(A(\mathbf{p}_{t+1})/A(\mathbf{p}_t))$, changes in the rate of time preference (ρ), and precautionary saving (σ_{t+1}^2). In particular, consumption growth is positively related to the difference between the real interest rate and time preference and to precautionary saving. The more impatient consumers are, the rate of time preference tends to be high relative to the interest rate, so the lower consumption growth and hence higher consumption. On the other hand, uncertainty leads consumers to defer consumption, to be more prudent. The more prudent consumers are, the higher consumption growth and hence lower current consumption and higher precautionary saving (see Caballero, 1995; Skinner, 1995). This suggests that the relationship between impatience and prudence determines consumers' optimal consumption pattern (see Carroll, 1997).

Equation (22) is identical in form to the log consumption Euler equation derived from the time-honored power utility function (see Deaton, 1992). Indeed, when $\eta = 1$, the two equations are (almost) exactly the same. However, in (22) real consumption growth is defined with specific price indexes derived from the demand system instead of the conventional method of deflating nominal consumption by the CPI. Moreover, the parameters in both the demand system and the Euler equation determine the growth of real consumption; hence our model captures the consumer's intratemporal as well as intertemporal decisions. In traditional Euler equation analyses, on the other hand, consumption growth reflects the consumer's intertemporal decisions only (see Hall, 1978).

It is well known that, with a power or isoelastic function for within-period preferences, the intertemporal elasticity of substitution is inversely linked to the degree of relative risk aversion; hence the intertemporal elasticity of substitution is often defined as the reciprocal of the elasticity of marginal utility of consumption (Blundell, Browning, and Meghir, 1994, and Attanasio and Weber, 1995). This result holds when within-period preferences are homothetic. However, with non-homothetic within-period preferences, the inverse relationship between intertemporal substitution and risk aversion is no longer valid. Hall (1988) argues that the estimate of the coefficient on the interest rate against consumption growth in a log-linearized Euler equation should be interpreted as the intertemporal elasticity of substitution and can only be informative about the degree of risk aversion under restrictive assumption. Since our model is characterized by non-homothetic preferences, it has the added capability of disentangling intertemporal substitution and risk aversion. In particular, from (22) we can derive the elasticity of intertemporal substitution in consumption (EIS) as

$$\text{EIS} = \frac{\partial \ln(C_{t+1} / C_t)^*}{\partial(r_{t+1} - \ln(A(\mathbf{p}_{t+1}) / A(\mathbf{p}_t))} = \frac{1}{2\eta + \zeta(2-\eta) - 1} . \tag{23}$$

From (18), the degree of relative risk aversion (RRA) is given by

$$RRA \equiv -\frac{\partial \ln \lambda_t}{\partial \ln C_t} = \frac{\zeta\left[1 + (1-\eta)\ln(C_t / A(\mathbf{p}_t))\right]}{\ln(C_t / A(\mathbf{p}_t))} - \frac{1 - 2\eta - \eta(1-\eta)\ln(C_t / A(\mathbf{p}_t))}{1 + (1-\eta)\ln(C_t / A(\mathbf{p}_t))} . \tag{24}$$

It is worth noting the role that η plays in determining EIS and RAA. When $\eta = 1$, EIS = $1/(1+\zeta)$ and RAA $= (\zeta / \ln(C_t / A(\mathbf{p}_t)) + 1$. The degree of relative risk aversion is a decreasing function of real income or consumption, which means that consumers are less risk averse as they reach a higher level of income or consumption. It is clear that EIS ≠ 1/RRA. Thus even under homothetic preferences, there is no specific relationship between risk aversion and intertemporal substitution in contrast to the traditional measure with the power utility function.

This discussion clearly reveals the linkage between commodity demands and intertemporal consumption. In particular, commodity demands in (143) depend on (log) total consumption, but consumption growth as given in (210) and hence consumption in turn is related to commodity prices, the interest rate, and precautionary saving behaviour. Thus we can investigate the effects of changes in commodity prices, the interest rate and precautionary saving on consumer demands through changes in consumption growth. This suggests that Marshallian or ordinary (uncompensated) price elasticities are not relevant in an intertemporal setting since they are derived under the condition that total expenditure is exogenously given. Consumer demand studies, however, treat the Marshallian price elasticities as relevant elasticities of demand.

C. Economic Issues

There are some economic issues that are relevant in analyzing consumer behavior. Firstly, according to (21), the same parameters regulate intertemporal substitution and precautionary saving. In fact, the degree of the intertemporal substitution elasticity is tied to – twice of – the degree of prudence associated with precautionary saving. Thus while our model allows risk aversion and intertemporal substitution to be separated, it is not capable of disentangling intertemporal substitution and precautionary saving. Our model, though, is a much improvement over existing results from the power utility function in which the same parameter controls risk aversion, intertemporal substitution, and precautionary saving. To distinguish the effects of intertemporal substitution and precautionary saving on consumption growth, we introduce an additional parameter to the conditional variance term by making it ½$(1+\kappa)$ where κ takes on a nonzero value. We can test whether $\kappa = 0$ or not to investigate the validity of equivalence of intertemporal substitution and precautionary saving effects.

Secondly, many studies found excess sensitivity of consumption to income (see Deaton, 1992, for reference). If there are binding borrowing constraints or capital market imperfections, consumption may be excessively sensitive to current income (see Jappelli and Pagano, 1989, for evidence). When capital markets are imperfect, the Euler equation (6) will not be satisfied, and there is substantial evidence for liquidity constraints (Zeldes, 1989, Campbell and Mankiw, 1990; Runkle, 1991). To test for excess sensitivity in consumption, we include income growth as an additional variable in (21). In contrast to earlier studies based on the Euler equation, we investigate excess sensitivity with the demand system and the Euler equation estimated jointly. Thirdly, habit formation is shown to have a significant effect on consumption behavior (Dynan, 2000), and we investigate whether its effect is significant in the joint framework of the demand system and the Euler equation.

Considering these points, the Euler equation we estimate here is

$$\begin{aligned}\Delta \ln C_{t+1} &= \theta_1 \left[r_{t+1} - \Delta \ln A(\mathbf{p}_{t+1}) - \rho \right] + \theta_2 \Delta \ln A(\mathbf{p}_{t+1}) + \theta_3 \Delta \ln G(\mathbf{p}_{t+1}) \\ &\quad + \theta_4 \Delta \ln H(\mathbf{p}_{t+1}) + \theta_5 \sigma_{t+1}^2 + \mu_Y \Delta \ln Y_{t+1} + \mu_C \Delta \ln C_t + \nu_{t+1} ,\end{aligned} \tag{25}$$

where $\theta_1 = 1/(2\eta + \zeta(2-\eta) - 1)$, $\theta_2 = (\eta + \zeta)\theta_1, \theta_3 = (\zeta - 1)\theta_1$, $\psi_4 = \eta(1-\zeta)\theta_1$, $\psi_5 = \theta_1(1+\kappa)/2$, $v_{t+1} = \theta_1 u_{t+1}$, and Y_{t+1} is disposable income at period t+1.

D. Econometric Issues

Equations (14) and (21) constitute a system of equations arising from the consumer's intratemporal and intertemporal optimization problems. We face some econometric issues in estimating this system of equations. Firstly, and most importantly, in the typical estimation of the demand system in consumer demand studies, consumption expenditure (typically referred to as "income") is treated exogenously. In an intertemporal context, consumption expenditure is an endogenous variable, making the demand system and the Euler equation a recursive system. Though recursive, however, separate estimation of these equations will not be efficient because there are cross-equation parameter restrictions that need to be imposed in estimation. In particular, there are parameters that are common in both the demand system and the Euler equation such as η and the parameters in $A(\mathbf{p}_t), B(\mathbf{p}_t)$, and $H(\mathbf{p}_t)$. Efficient estimation then requires joint estimation of the demand system and the Euler equation with cross-equation parameter restrictions imposed. Blundell, Browning, and Meghir(1994) and Attannasio and Weber (1995), in contrast, estimate the AIDS demand system and derive the estimate of the intertemporal elasticity of substitution occurring in the Euler equation, conditional on within-period demand system parameters.

Secondly, for aggregate time series data, endogeneity of the right hand variables such as prices and the interest rate is bound to occur in estimating (14) and (21). However, finding true instruments may be difficult because the instruments may not be completely exogenous. Moreover, in this study we are more interested in qualitative results than in precise estimation of specific parameters. Thus we ignore the endogeneity problem in estimation. Thirdly, consumption growth depends on precautionary saving, and it is the conditional, not unconditional, variance of consumption growth innovation (σ_{t+1}^2) that determines precautionary saving. Dynan (2000), instead, uses the unconditional variance of consumption growth as a measure of precautionary saving. Other studies (e,g., Guiso, Jappelli, and Terlizzese, 1992) employ the variance of income growth as a measure of consumption risk. We use the ARCH (Autoregressive Conditional Hetroskedasticity)-type method to estimate the conditional variance of consumption growth innovation.

Fourthly, for time series data, autocorrelation is a serious problem. Berndt and Savin (1975) show that, for share equations subject to autocorrelation, inclusion of lagged own shares is compatible with the adding-up constraint only if the first-order autocorrelation coefficient is the same for all equations. Their method has been widely employed in many empirical studies but has no theoretical underpinning. We derive a model consistent basis for including lagged shares in the demand system. We do this by modelling path dependence of the behaviour of the consumer, extending the definition of the price index $\ln A(\mathbf{p}_t)$ in (13) to include an additional term that is a function of lagged commodity shares. A detailed discussion is provided in the Appendix.

III. ESTIMATION AND RESULTS

A. Data

The data set of this study relates to six broad groupings out of seventeen private final consumption expenditures in Australia for 1960/61 - 2001/02, taken from the Australian Bureau of Statistics' *Australian National Accounts*. The six categories are (1) cigarettes & tobacco, and alcoholic beverages, (2) food & non-alcoholic beverages, clothing and footwear, electricity, gas & other fuel, and health, (3) furnishings & household equipment, (4) purchase of vehicles, operation of vehicles, and transport services, and (5) rent & other dwelling services, communications, recreation & culture, education services, hotels & cafes & restaurants, and insurance & other financial services, and (6) other goods and services.[1] The price series of different commodities are their implicit price indices (setting all prices in 1979/80 = 1) obtained by dividing the current price series by the corresponding constant price series. The seventeen commodities are then aggregaged into six groups. Expenditures are then summed, and group price indexes are formed using Divisia indexes using the individual price deflators and the averages of current and lagged shares as weights. Implied quantities for the six groups are then formed as deflators and shares are constructed. Total consumption is the sum of expenditures on the six categories. Disposable income is personal income minus personal income tax. The interest rate is taken to be the annual average home loan rate in the banking industry.

B. Estimation Results

We have demonstrated that efficient estimation requires that the demand system and the Euler equation need to be estimated jointly. Our primary aim is to evaluate efficiency gain when the demand system and the Euler equation are estimated jointly as opposed to when they are estimated separately or sequentially.[2]

Table 1 presents results of estimating the share and Euler equations separately. Table 1a shows that all the first-order terms of the price effects (α's and β's) are significant at conventional levels of significance, but the second-order terms (γ's) are not. Interestingly, the value of η is 0.5976 and significant. When its value is 1, the MAIDS is effectively the AIDS. This indicates that despite its wide use, the AIDS is restrictive relative to the MAIDS in estimating the demand system. The results of estimating the Euler equation were not successful. η is unidentified and AIDS versus MAIDS cannot be distinguished. Estimating some slightly more complex variations (for example with lags) is sufficient to get identification technically, but results are not robust to alternative specifications. In most cases, income growth appears significant in the Euler equation when estimated with no precautionary saving but this disappears when precautionary saving is allowed. Table 1b presents estimation results for the Euler equation by setting η to 1.0 and the second-order terms of the price effects zero. Four squared consumption growth innovation terms are added to account for precautionary saving effects. Several things are worth noting. ζ has a high value and is significant, underscoring the significance of intertemporal effects in consumption. Income growth and lagged consumption growth terms are insignificant. There is, however, some evidence of precautionary saving. The degree of the intertemporal elasticity of substitution is 0.1484, and the value of relative risk aversion is 2. The coefficients for the price effects in the Euler equation are drastically different from those estimated from the share equations. The theoretical validity, however, requires that they should be the same because the two equations are derived from the same optimizing model. Thus separate estimation results are questionable.

Blundell, Browning, and Meghir (1994) and Attannasio and Weber (1995) employ a sequential method to estimate the demand system and the Euler equation. They first estimate the demand system and generate the price index for in $A(\mathbf{p}_t), B(\mathbf{p}_t)$, and $H(\mathbf{p}_t)$. They then use these price indexes to estimate the parameters of the Euler equation, especially the degree of intertemporal substitution. Sequential estimation should help improve efficiency in estimating the Euler equation relative to separate estimation because it uses prior information. Table 2 presents sequential estimation results using the parameters of the share equation in Table 1a. The results are sensitive whether or not precautionary saving is allowed for. ζ is significant without or with precautionary saving. Income growth and lagged consumption growth are significant under no precautionary saving but insignificant under precautionary saving. The precautionary saving effect, however, appears strong but insignificant. When precautionary saving is allowed, the elasticity of intertemporal substitution is 0.8685 and the value of relative risk aversion is 0.1952.

Table 3 shows joint estimation results with and without precautionary saving. Four squared consumption growth innovation terms are added to account for precautionary saving. The coefficients for price effects without and with precautionary saving are almost the same. η and ζ are both significant, underscoring their role in consumer demand and intertemporal consumption. As in separate estimation, the value of suggests that the AIDS is restrictive relative to the MAIDS in modeling the consumer demand system. Joint estimation without precautionary saving shows that income growth is insignificant, but results with precautionary saving show that neither income growth nor lagged consumption growth are significant. There is some evidence of a moderate but significant precautionary saving effect. These suggest that excess sensitivity and habit formation effects are not significant; hence life-cycle and precautionary saving motives are reasonable to characterize the aggregate Australian consumption behavior. When precautionary saving is allowed, the elasticity of intertemporal substitution is 0.2375 and the value of relative risk aversion is 0.3136. Joint estimation should improve efficiency gain for both the demand system and the Euler equation. Comparing Table 1a and Table 3, efficiency gains for share equations appear small. From Table 1b, Table 2 and Table 3, on the other hand, we can see that joint estimation clearly helps improve efficiency gain for the Euler equation. We should, however, expect that joint estimation results are more reliable than separate or sequential estimation results since it incorporates common theoretical restrictions on both the demand system and the Euler equation.

IV. CONCLUSION

This paper has examined the linkage between consumer demands and intertemporal consumption and investigated the relevance of the traditional approach to consumer behavior. While there are scores of studies on consumer demand and intertemporal consumption, they are conducted independent of each other. This paper represents the first attempt to fully integrate the demand system and the Euler equation. We have found that joint estimation helps improve efficiency gain for the Euler equation, but efficiency gains for the demand system are small.

APPENDIX

We derive a model consistent basis for including lagged shares in the demand system. We do this by modelling path dependence of the behaviour of the representative consumer, extending the definition of the price index $\ln A(\mathbf{p}_t)$ to include an additional term which is a function of lagged commodity shares. We require the additional term to be homogeneous of degree zero in prices and total expenditure in order to preserve homogeneity of degree zero of the indirect utility function. Consequently our revised price index, $\ln P(\mathbf{p}_t)$ below, is no longer itself homogeneous of degree one in prices. This can be interpreted as introducing a scale effect into preferences. It results in the inclusion of lagged shares in the demand system.

Our generalization of the price index (13) is

$$\ln P(C_t, \mathbf{p}_t, \mathbf{s}_{t-1}) = \ln A(\mathbf{p}_t) + \sum\nolimits_{k=1}^{n} \omega_k \ln(S_{k,t-1} / S_{k0}) \ln(p_{kt} / C_t), \tag{A1}$$

where $S_{k,t-1}$ denotes commodity share k at time $t-1$ and $\mathbf{s}_{t-1}$ denotes the vector of n such shares. With this extension, the MAIDS indirect utility function (10) generalizes to

$$V^*(C_t, \mathbf{p}_t, \mathbf{s}_{t-1}) = \frac{C_t^{1-\eta}}{G(\mathbf{p}_t) H(\mathbf{p}_t)^{-\eta}} \ln\left(\frac{C_t}{P(C_t, \mathbf{p}_t, \mathbf{s}_{t-1})} \right), \tag{A2}$$

which can be written more conveniently as

$$V^*(C_t, \mathbf{p}_t, \mathbf{s}_{t-1}) = \frac{C_t^{1-\eta}}{G^*(\mathbf{p}_t) H(\mathbf{p}_t)^{-\eta}} \ln\left(\frac{C_t}{A^*(\mathbf{p}_t, \mathbf{s}_{t-1})} \right), \tag{A3}$$

where $G^*(\mathbf{p}_t, \mathbf{s}_{t-1}) = G(\mathbf{p}_t)/(1+\pi(\mathbf{s}_{t-1}))$, $\pi(\mathbf{s}_{t-1}) = \sum\nolimits_{k=1}^{n} \omega_k \ln(S_{k,t-1} / S_{k0})$ and $\ln A^*(\mathbf{p}_t, \mathbf{s}_{t-1})$ $= \ln A(\mathbf{p}_t) + \sum\nolimits_{k=1}^{n} \omega_k \ln(S_{k,t-1} / S_{k0}) \ln(p_{kt})$. There are no adding up restrictions on the ω_k's.[3] However, interpretation is enhanced by imposing monotonicity restrictions $\omega_k \geq 0$.[4] In order to interpret the generalized indirect utility function (A3), it is most straightforward to first exhibit the share equation system that follows from it by Roy's identity. For this purpose, we assume that the representative agent is not aware of the impact of individual decisions on the price index through the lagged share effects, even though the preferences of the agent are influenced by the lagged shares. We are thus pursuing the private optimum rather than the social optimum, in order ultimately to confront the model with data associated with the behaviour of private agents. The implied demand system is

$$S_{it}=\frac{\alpha_i+\sum_{j=1}^{n}\gamma_{ij}\ln p_{jt}+\omega_i\ln(S_{i,t-1}/S_{i0})+(\beta_i-\eta\alpha_i)\ln(C_t/A^*(\mathbf{p}_t,\mathbf{s}_{t-1}))}{1+\sum_{j=1}^{n}\omega_j\ln(S_{j,t-1}/S_{j0})+(1-\eta)\ln(C_t/A^*(\mathbf{p}_t,\mathbf{s}_{t-1}))},\tag{A4}$$

The commodity share system is fractional and may be interpreted as a generalization of (14). By redefining an extended set of weights as

$$z_{1,t}=\frac{1}{1+(1-\eta)\ln\left(C_t/A^*(\mathbf{p}_t,\mathbf{s}_{t-1})\right)+\sum_{k=1}^{n}\omega_k\ln\left(S_{k,t-1}/S_{k0}\right)}$$

$$z_{2,t}=\frac{(1-\eta)\ln\left(C_t/A^*(\mathbf{p}_t,\mathbf{s}_{t-1})\right)}{1+(1-\eta)\ln\left(C_t/A^*(\mathbf{p}_t,\mathbf{s}_{t-1})\right)+\sum_{k=1}^{n}\omega_k\ln\left(S_{k,t-1}/S_{k0}\right)}$$

$$z_{3,it}=\frac{\ln\left(S_{i,t-1}/S_{i0}\right)\sum_{k=1}^{n}\omega_k}{1+(1-\eta)\ln\left(C_t/A^*(\mathbf{p}_t,\mathbf{s}_{t-1})\right)+\sum_{k=1}^{n}\omega_k\ln\left(S_{k,t-1}/S_{k0}\right)}$$

the share system can be rewritten more intuitively as

$$S_{it}=z_{1,t}\chi_{it}+z_{2,t}\psi_i+z_{3,it}\tau_i\tag{A5}$$

where $\chi_{it}=\alpha_i+\sum_{j=1}^{n}\gamma_{ij}\ln p_{jt}$, $\psi_i=(\beta_i-\eta\alpha_i)/(1-\eta)$, and $\tau_i=\omega_i/\sum_{k=1}^{n}\omega_k$.

In interpreting this extended system, note that the $z_{3,it}$ are commodity specific and that $\sum_{i=1}^{n}z_{3,it}\tau_i=1-z_{1,t}-z_{2,t}$. We note also that $S_{i,t-1}/S_{i0}=1$ for time period $t=1$. We also scale the data so as to normalize on period 1, viz. $C_t=p_{jt}=1$ for all j at time $t=1$. Hence $C_t/A^*(\mathbf{p}_t,\mathbf{s}_{t-1})=1$ at $t=1$. Thus $z_{2,t}=z_{3,it}=0$ and $z_{1,t}=1$ for $t=1$ and $S_{it}=\chi_{it}=\alpha_i$ at $t=1$. Asymptotic behavior of (A5) is influenced by the weight $z_{2,t}$, with $S_{it}\to\psi_i$ as real income expands indefinitely. Were it not for the presence of the parameter τ_i scaled by the commodity specific weights $z_{3,it}$ in (A5), the movement of shares from α_i for low expenditure to ψ_i for asymptotically high expenditure would be monotonic. However, (A5) displays considerably greater complexity as time passes and circumstances change due to the weight given to the ‘revealed preference’ term τ_i. With $\omega_k>0$ as part of the specification, $z_{3,it}$ will be negative if the immediately previous share is below the 'time zero' share. In this case, since $\tau_i > 0$, the term $z_{3,it}\tau_i$ reduces the predicted share. Conversely the predicted share is raised if $z_{3,it}$ is positive. In principle, this specification can introduce non-linearity in shares as a function of real total expenditure allowing a greater degree of flexibility in empirical fitting. The β_i parameters are interpretable as intermediate shares, representing predicted shares when real expenditure equals $1/\eta$ (abstracting from the influence of lagged shares).

Noting the comparability of (A3) with the initial specification (A1), our empirical specification for the Euler equation associated with the system is obtained by rewriting (25) at time t as

$$\ln\left(\frac{C_t}{C_{t-1}}\right) = \theta_1\left[\left(r_t - \ln\left(\frac{A^*(\mathbf{p}_t,\mathbf{s}_{t-1})}{A^*(\mathbf{p}_{t-1},\mathbf{s}_{t-2})}\right)\right) - \rho\right] + \theta_2 \ln\left(\frac{A^*(\mathbf{p}_t,\mathbf{s}_{t-1})}{A^*(\mathbf{p}_{t-1},\mathbf{s}_{t-2})}\right) + \theta_3 \ln\left(\frac{G^*(\mathbf{p}_t,\mathbf{s}_{t-1})}{G^*(\mathbf{p}_{t-1},\mathbf{s}_{t-2})}\right) + \theta_4 \ln\left(\frac{H(\mathbf{p}_t)}{H(\mathbf{p}_{t-1})}\right) + \theta_5\sigma_t^2 + \nu_t. \tag{A6}$$

where, for convenience, we collect the modified price indexes as [5]

$$\ln A^*(\mathbf{p}_t,\mathbf{s}_{t-1}) = \sum_{i=1}^n \alpha_i \ln p_{it} + \tfrac{1}{2}\sum_{i=1}^n \sum_{j=1}^n \gamma_{ij} \ln p_{it} \ln p_{jt} + \sum_{i=1}^n \omega_i \ln(S_{i,t-1}/S_{i0})\ln(p_{it})$$

$$\ln G^*(\mathbf{p}_t,\mathbf{s}_{t-1}) \doteq \sum_{i=1}^n \beta_i \ln p_{it} - \sum_{i=1}^n \omega_i \ln(S_{i,t-1}/S_{i0})$$

$$\ln H(\mathbf{p}_t) = \sum_{i=1}^n \alpha_i \ln p_{it}$$

Equality restrictions are $\sum_{i=1}^n \alpha_i = 1$, $\sum_{i=1}^n \beta_i = 1$ (adding up) and $\gamma_{ij} = \gamma_{ji}$ (symmetry). These conditions are imposed in estimation. Additionally, reasonable positivity restrictions are $\alpha_i > 0,\ \beta_i > 0,$ and $\omega_i > 0.$ These restrictions are not imposed in estimation but hold for all preferred results.

FOOTNOTES

[1] The six groups were determined by searching over up to nine possible groupings and combining categories that gave best results using lack of autocorrelation in share equation estimates as the criterion. (Basically the criterion was just a DW statistic from the regression of the actual on predicted shares.) Thus some of these commodity grouping might appear odd in terms of the categories of commodities they contain, but they are "model consistent" aggregations in the sense that the aggregated shares have "optimal" properties in terms of their residual characteristics.

[2] It is often difficult to credibly estimate subjective discount rates, and estimation of the models by varying the discount rate does not provide satisfactory results; hence we fix its value at 5 percent (see Cagetti, 2001).

[3] The existence of a scale effect can be tested by imposing the restriction $\sum_{k=1}^{n} \omega_k = 0$. Evidence that such a restriction is empirically refuted is available in the statistical significance of the freely estimated ω_k.

[4] Restricting the ω_k to be positive is not necessary, but it does appear to be natural as it implies sluggish first order monotonic adjustment of shares over time. Free estimation almost invariably produces positive ω_k.

[5] See the definitions of $\ln A^*$ and $\ln G^*$ following (A3). The empirical specification for $\ln G^*$ uses the approximation $\ln(1+\pi) \doteq \pi$.

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TABLE 1a SEPARATE ESTIMATION RESULTS: SHARE EQUATIONS (Asymptotic t Ratios in Parentheses)

Parameter	Estimate	Parameter	Estimate	Parameter	Estimate
α_2	0.2635 (305.12)	α_3	0.0851 (114.55)	α_4	0.1310 (155.68)
α_5	0.4084 (369.67)	α_6	0.0579 (153.85)	β_2	0.2325 (24.80)
β_3	0.0712 (12.88)	β_4	0.1271 (19.61)	β_5	0.4638 (28.19)
β_6	0.0562 (17.38)	γ_{22}	0.0339 (2.32)	γ_{23}	-0.0014 (-0.22)
γ_{24}	-0.0072 (-0.94)	γ_{25}	-0.0312 (-2.19)	γ_{26}	-0.0112 (1.55)
γ_{33}	-0.0074 (0.97)	γ_{34}	0.0026 (0.44)	γ_{35}	-0.0076 (-1.08)
γ_{36}	0.0057 (1.02)	γ_{44}	0.0244 (2.23)	γ_{45}	-0.0048 (-0.48)
γ_{46}	-0.0099 (-2.40)	γ_{55}	0.0479 (2.30)	γ_{56}	-0.0043 (-0.72)
γ_{66}	-0.0082 (-1.19)	η	0.5976 (4.99)	θ_1	0.0439 (21.88)
θ_2	0.2145 (14.63)	θ_3	0.0568 (14.61)	θ_4	0.1083 (10.89)
θ_5	0.4381 (10.45)	θ_6	0.0656 (15.61)		

TABLE 1b SEPARATE ESTIMATION RESULTS: EULER EQUATION
(Asymptotic t Ratios in Parentheses)

Parameter	Estimate	Parameter	Estimate	Parameter	Estimate
α_2	-0.7187 (-0.11)	α_3	6.8206 (1.63)	α_4	-0.5966 (-0.21)
α_5	-12.3530 (-1.79)	α_6	1.9012 (0.43)	β_2	0.4456 (0.55)
β_3	-0.4902 (-0.56)	β_4	0.2767 (0.77)	β_5	1.5678 (0.95)
β_6	-0.0413 (-0.06)	ζ	5.7377 (4.28)	κ_1	2.1668 (1.77)
κ_2	-1.2360 (-0.91)	κ_3	1.3530 (1.13)	κ_4	0.7445 (0.69)
θ_1	0.2409 (3.08)	θ_2	1.1640 (3.02)	θ_3	0.2888 (3.03)
θ_4	0.7059 (3.06)	θ_5	2.6277 (3.08)	θ_6	0.2807 (3.20)
μ_I	0.0278 (1.58)	μ_C	0.0277 (0.24)		

TABLE 2 SEQUENTIAL STIMATION RESULTS: EULER EQUATION
(Asymptotic t Ratios in Parentheses)

Parameter	Estimate	Parameter	Estimate	Parameter	Estimate
		No Precautionary Saving			
ζ	2.7920 (10.40)	μ_I	0.0068 (2.13)	μ_C	0.2993 (2.84)
		Precautionary Saving			
ζ	3.1446 (8.17)	κ_1	0.5690 (0.80)	κ_2	0.7511 (1.11)
κ_3	1.5075 (2.03)	κ_4	0.7131 (1.01)	μ_I	0.12E-01 (0.54)
μ_C	0.1775 (1.71)				

TABLE 3a JOINT ESTIMATION RESULTS: SHARE AND EULER EQUATIONS WITH NO PRECAUTIONARY SAVING
(Asymptotic t Ratios in Parentheses)

Parameter	Estimate	Parameter	Estimate	Parameter	Estimate
α_2	0.2640 (372.09)	α_3	0.0864 (159.95)	α_4	0.1300 (183.63)
α_5	0.4094 (480.79)	α_6	0.0576 (171.17)	β_2	0.2276 (24.80)
β_3	0.0713 (13.18)	β_4	0.1324 (23.69)	β_5	0.4655 (30.42)
β_6	0.0544 (16.73)	γ_{22}	0.0458 (3.21)	γ_{23}	-0.0056 (-0.91)
γ_{24}	-0.0137 (-1.80)	γ_{25}	-0.0373 (-2.84)	γ_{26}	0.0164 (2.39)
γ_{33}	-0.0014 (0.18)	γ_{34}	0.0056 (0.88)	γ_{35}	-0.0055 (-0.82)
γ_{36}	0.0040 (0.76)	γ_{44}	0.0456 (3.77)	γ_{45}	-0.0243 (-2.43)
γ_{46}	-0.0107 (-2.43)	γ_{55}	0.0719 (3.56)	γ_{56}	-0.0069 (-1.28)
γ_{66}	-0.0076 (-1.20)	η	0.5972 (5.71)	ζ	2.5084 (12.22)
μ_I	-0.0021 (0.78)	μ_C	0.1630 (1.92)	θ_1	0.0456 (26.95)
θ_2	0.2167 (15.29)	θ_3	0.0628 (18.42)	θ_4	0.0998 (11.22)

(Continued on next page)

TABLE 3a - CONTINUED

Parameter	Estimate	Parameter	Estimate	Parameter	Estimate
θ_5	0.3931 (10.05)	θ_6	0.0662 (16.55)		

TABLE 3b JOINT ESTIMATION RESULTS: SHARE AND EULER EQUATIONS WITH PRECAUTIONARY SAVING
(Asymptotic t Ratios in Parentheses)

Parameter	Estimate	Parameter	Estimate	Parameter	Estimate
α_2	0.2641 (289.54)	α_3	0.0844 (116.60)	α_4	0.1301 (166.61)
α_5	0.4091 (344.61)	α_6	0.0581 (142.67)	β_2	0.2313 (25.56)
β_3	0.0730 (13.93)	β_4	0.1383 (22.60)	β_5	0.4529 (27.84)
β_6	0.0558 (16.52)	γ_{22}	0.0368 (2.61)	γ_{23}	-0.0029 (-0.45)
γ_{24}	-0.0159 (-1.93)	γ_{25}	-0.0261 (-1.87)	γ_{26}	0.0126 (1.76)
γ_{33}	0.0082 (1.11)	γ_{34}	0.0004 (0.07)	γ_{35}	-0.0083 (-1.20)
γ_{36}	0.0046 (0.85)	γ_{44}	0.0457 (3.85)	γ_{45}	-0.0169 (-1.65)
γ_{46}	-0.0105 (-2.43)	γ_{55}	0.0500 (2.33)	γ_{56}	-0.0009 (-0.17)
γ_{66}	-0.0114 (-1.63)	η	0.6568 (5.50)	ζ	2.9002 (9.83)
κ_1	0.8750 (1.83)	κ_2	0.1939 (0.41)	κ_3	1.0369 (2.21)
κ_4	0.3326 (0.73)	μ_I	-0.0047 (-0.25)	μ_C	0.0785 (0.93)

(Continued on next page)

TABLE 3b - CONTINUED

Parameter	Estimate	Parameter	Estimate	Parameter	Estimate
θ_1	0.0434 (22.02)	θ_2	0.2058 (15.26)	θ_3	0.0536 (14.14)
θ_4	0.0982 (11.13)	θ_5	0.4581 (10.95)	θ_6	0.0655 (15.12)