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Enhancing teachers' knowledge of students' thinking:

The case of graphics calculator graphs.

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Graphics calculators are widely available in Australian schools, particularly as a tool for drawing and interpreting graphs in mathematics instruction. There is much research on pedagogical practices associated with graphics calculators, but relatively little on the design of professional development programs involving the use of graphics calculators. What calculator knowledge do teachers need? And would informing teachers about how students interpret the graphics calculator display lead to better student outcomes?

This paper reports on a graphics calculator in-service program based on the principles of Cognitively Guided Instruction (CGI). A series of clinical interviews identified student misconceptions associated with interpreting straight-line graphs and parabolas on a graphics calculator. Interview results were reported to teachers during a two-day workshop. Teachers were subsequently observed using graphics calculators in the classroom and students from these classes were interviewed using the original protocols.

Results show that the CGI intervention was largely successful. Teachers reported greater confidence in using the technology in the classroom and dealt with a wide variety of examples that confronted student misconceptions. Students from the observed classes performed significantly better than the control group on tasks that required them to interpret graphical images on the graphics calculator screen.

INTRODUCTION

Graphics calculators were first developed in the mid 1980s and since then their use in secondary classrooms has become widespread across Australia. Although the second and third generation graphics calculators available in schools today possess a wide array of mathematical features, their ability to display graphs quickly and link them to symbolic and numerical representations of functions remains their primary use in mathematics instruction.

There has been much research on the use of graphics calculators as tools in the teaching and learning of mathematics in the secondary school (Dunham & Dick, 1994). However, very little of this research has been able to provide any clear directions on how to maximise the benefits of the technology in the classroom (Penglase & Arnold, 1996). Most of the studies in the literature are essentially teaching experiments comparing pedagogical practices in experimental classes, where graphics calculators are made available, and control groups where there is no use of the technology (e.g. Goos et al., 2000; Harskamp, Suhre, & Van Struen, 2000; Simonsen & Dick, 1997; Farrell, 1996). These studies tend to focus on changes in the dynamic nature of teacher and student roles in the presence of graphics calculators. Results suggest that graphics calculators can be associated with the emergence of more collaborative learning, greater use of open-ended questioning styles, and an increase in exploration of new concepts by students as opposed to a traditional, teacher-centered exposition.

But the findings of other studies are less conclusive and suggest that the picture is slightly more ambiguous. Simmt (1997) observed six high school teachers and noted that they taught essentially the same kinds of activities whether using graphics calculators or not. She proposed that merely providing the tool would have little impact on instruction unless teachers were given new strategies to help them reformulate their classroom practices in order to take full advantage of the technology.

Tharp, Fitzsimmons and Ayers (1997) investigated how 261 teachers used graphics calculators in the classroom. They found that teachers who viewed mathematics as rule-based were less likely to use graphics calculators for student investigations and more likely to think that graphics calculators did not enhance instruction. Tharp, Fitzsimmons and Ayers (1997) suggest that when teachers reflect on their current practice, share their insights with colleagues, and design new curriculum materials they can move towards changing their instructional approach and begin to use graphics calculators in more productive ways.

Doerr and Zangor (2000) report that the degree to which teachers understand some of the basic operations of the graphics calculator, such as how it highlights and joins discrete pixels to produce a graph, is also critical in assisting teachers make better use of the machine. Teachers who are confident in their own knowledge of the technology deal better with the unexpected calculator displays that inevitably arise and are more likely to encourage their students to analyse critically the calculator output.

Clearly the graphics calculator has much to offer the secondary mathematics teacher, but it is essential that carefully designed support structures are in place if the real benefits of the technology are to be realised. The present study focuses on the evolution of a professional development model designed to educate teachers in the use of graphics calculators in two aspects: learning how to operate a graphics calculator, and learning how to use one effectively in the classroom.

THEORETICAL FRAMEWORK

This research is informed and conceptualized within the current literature on teacher professional development, and more specifically within the framework of the program known as Cognitively Guided Instruction (CGI). Key features of the CGI model are outlined next, with particular emphasis on how the principals of CGI relate to the present study.

Cognitively guided instruction

CGI was developed by researchers at the University of Wisconsin in the 1980s as a means of assisting elementary teachers in developing more efficient problem solving strategies in simple addition and subtraction contexts (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989). CGI is an approach to mathematics professional development and classroom pedagogy that has two basic tenets: first, that instruction should be guided by a teacher's insight into what students know; and secondly, students best learn mathematics by solving problems (Fennema, Carpenter & Peterson, 1989).

Typically, CGI professional development sessions inform teachers about research data that describes well-defined taxonomies of particular kinds of mathematical problems and solution strategies. Teachers discuss the implications of the research and then work collaboratively to prepare lessons. The lesson plans are discussed and teachers receive feedback from their peers. The CGI model is not prescriptive and there is no formal curriculum or set of instructional materials provided to teachers. CGI teachers are not trained to do certain things or use particular teaching tools. Instead, they are provided with general principles for assessing students' current thinking. Each teacher is then free to design a mathematics program suited to the specific needs of his or her students (Franke & Kazemi, 2001).

CGI professional development workshops demonstrate how teachers can assess students' thinking by presenting research-based summaries of how students typically solve certain kinds of problems and indicating likely student misconceptions associated with these problem types. On-going classroom support is also made available to teachers so that they can use knowledge about their students in making instructional decisions and fostering a rich understanding of mathematical concepts (Fennema, Carpenter, Franke, Levi, Jacobs & Empson, 1996).

CGI pedagogy focuses on what students know rather than on what teachers do (Carpenter , Franke, Levi, & Empson, 1999). As such, CGI teaching adopts a constructivist perspective by promoting the belief that students are responsible for their own learning. Students solve problems using methods of their own choosing and in ways that make sense to them. Teachers refrain from showing students what to do and allow them to resolve issues among themselves. The social aspects of learning are also emphasized: Students work cooperatively, discuss the procedures they have used, and report them to the class for analysis and validation (Hiebert, Carpenter, Fennema, Fusin, Wearne, Murray, Olivier & Human, 1997).

The CGI classroom is therefore characterised by students who are engaged in the solution of challenging, non-routine problems. Teachers use their knowledge of students' thinking to choose problems that are commensurate with their students' abilities, and to formulate questions that further elicit students' understanding. Armed with this information, teachers can direct students to solve the specific problem types they are ready to learn about, and to structure new problems that challenge students to advance their mathematical thinking (Carpenter, Fennema & Franke, 1996).

Previous CGI studies

As noted already, CGI originated in a study about the teaching of addition and subtraction word problems in Years 1 and 2 (Carpenter et al., 1989) and most of the early CGI literature relates to research carried out in the beginning primary grades. However, the CGI methodology has been applied successfully in subsequent studies in other grades and in work with preservice teachers. Bright and Vacc (1994) developed a CGI program for preservice primary teachers designed to change the undergraduates' beliefs and perceptions about the nature of mathematics instruction. They hypothesised that these changes in attitude would, in turn, improve the teaching performance of the trainee teachers.

Bright and Vacc (1994) found that the CGI model was effective in shifting the preservice teachers' view of mathematics pedagogy from a fairly traditional paradigm, which focused on helping children master mathematical concepts and procedures modeled by the teacher, to one aligned more closely to constructivism. They found that CGI processes also assisted the preservice teachers develop more effective ways of planning instruction based on data gathered about each student's thinking. However, Bright and Vacc (1994) also reported that the preservice teachers were unable to maintain the progress they had made once the support systems associated with the CGI intervention were withdrawn.

Tirosh (2000) also developed a teacher education course based on CGI. The unit of study developed by Tirosh (2000) considered the teaching and learning of rational numbers in the primary grades and included a strong focus on familiarising the preservice teachers with the common thought processes used by children when they operate on fractions. She concluded that informing preservice teachers about likely student errors and misconceptions was a significant factor in improving classroom instruction and recommended that such information should form an integral part of future teacher training in this area.

Anthony, Bicknell and Savell (2001) showed how a CGI-based professional development numeracy program could lead to long-lasting changes in teachers' classroom practice. In particular, they noted the value in shifting the focus from teacher behaviour to student thinking that lies at the heart of CGI. Teachers whose primary concern was how they would implement the numeracy program were not as successful as those whose main emphasis was on learning about their students' numerical thought processes.

These studies validate many of the positive results of the original CGI research. They confirm the importance of teacher knowledge as a critical component of the CGI model. They reinforce the emphasis CGI places on students' thinking, and on a close examination of likely student errors and misconceptions in particular, as the basis for instruction. And they demonstrate that CGI professional development processes can be applied successfully to change teachers' attitudes about the nature of mathematics instruction, and that this can lead to more favourable classroom outcomes.

The present study applies the principles of CGI professional development and classroom practice in a new context, the use of graphics calculator technology in pre-calculus secondary mathematics teaching.

METHOD

Phase One

CGI professional development programs begin with an analysis of students' thinking. So it was first necessary to gather data on how students typically interpret the visual images they see displayed on the graphics calculator screen and the meanings that students attach to some basic calculator operations like re-setting the view window and zooming. This became the focus of Phase One of the present study.

Participants

A series of clinical interviews were conducted with twenty-five students, five from each of five metropolitan high schools in Sydney (13 girls and 12 boys). The students were all studying the highest level mathematics course available and, although they had previously used a graphics calculator (the Casio *fx-7400G*), they could best be described as novice users of the technology.

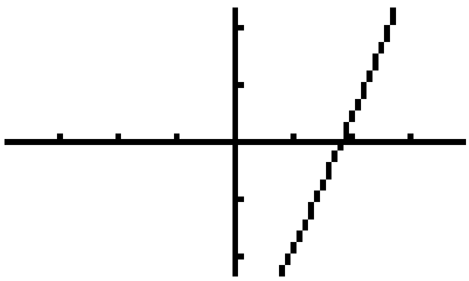
At the time of the interviews, the students were pre-calculus and had studied linear and quadratic functions and their graphs. They had used graphics calculators to plot straight lines and parabolas, and to read off the values of critical points (such as intercepts and roots) from the calculator screen. The students could also operate some of the basic graphing features of the calculator (tracing, zooming and scrolling the view window).

Student Interviews

The students were interviewed individually for about fifty minutes on three separate occasions, approximately two weeks apart. During each interview, the student was asked to consider two or three tasks that were specifically designed to probe their thinking as they interpreted the calculator graphs. The series of eight interview tasks created problematic situations for students because they exposed some of the technical limitations of the technology (e.g. the relatively low resolution of the screen and the difficulties associated with representing the graph of a continuous function by a series of discrete pixels). The students were asked to explain in-depth the unusual graphical images they saw and to resolve any inconsistencies between the graphs they had expected to see and those produced by the calculator. A sample of two of the student interview tasks, with instructions and initial calculator screens, is shown in Figure 1.

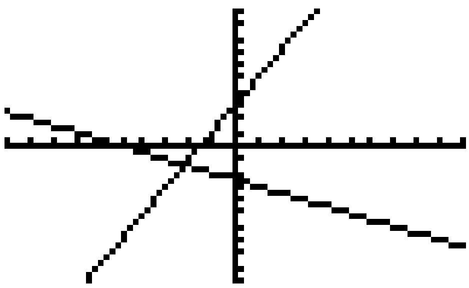
The first calculator screen in Figure 1 shows the graph of $y = 0.1x^2 + 2x - 4$ in the initial viewing window. The default viewing window displays only a partial view of the parabola and the image appears as a straight line rather than the U-shaped curve typically associated with a quadratic function of this type. The task investigated whether students would refer to the symbolic expression and use their knowledge of quadratic functions to recognise the incomplete graph, and what they did to obtain a more representative image.

The second screen in Figure 1 shows the graphics calculator window reset to display values from -10 to 10 on both axes. The coordinate axes are therefore unequally scaled in the rectangular viewing window and the two lines do not appear at right angles on the screen as one would expect from observing their gradients. The task explored how well students understood the effect of scaling the two axes differently, and what they thought the calculator was doing when they reconfigured the window parameters by zooming or scrolling, for instance.



Draw a sketch of $y = 0.1x^2 + 2x - 4$.

You may use the graphics calculator to help you.



Explain why the graphs of the lines $y = 2x + 3$ and $y = -0.5x - 2.5$ do not appear at right angles on the screen.

What could you do to make the lines look more perpendicular?

Figure 1. Instructions and initial calculator screens for two of the interview tasks.

Analysis of the student interview responses confirmed that many errors were due to an incomplete understanding of some fundamental mathematical concepts—such as making links between different representations of functions, dealing with unequal scales, and issues related to decimal approximations of irrational numbers. Misconceptions also arose because the students did not have a satisfactory understanding of what the calculator was doing when they used its zoom facility, nor did they recognise the processes used to assign coordinate values to the pixels that comprise the screen.

The students interviewed in Phase One of the study also acted as a control group against which the results of the Phase Two student sample could be compared. A complete summary of the interview tasks and results can be found in an earlier report of Phase One (Mitchelmore & Cavanagh, 2000).

Phase Two

Traditional professional development programs in the use of graphics calculators generally instruct teachers in the basic operations of the machine and provide ready-made worksheets and other resources for classroom instruction. However, there is considerable evidence that this approach has only limited success (Waits & Demana, 2000) because it fails to take account of the broader pedagogical issues associated with the introduction of the technology (Goos et al., 2000). For example, there is usually no discussion about how students interpret calculator graphs, nor are likely student errors and misconceptions associated with graphics calculators explored. In contrast, the CGI model, with its emphasis on problem solving activities that are developed using knowledge of students' thinking, appeared to offer a better alternative. So a workshop for teachers, based around the results of the earlier student interviews, became the focus of Phase Two of the present study.

The basic hypothesis of Phase Two is that a workshop modeled on CGI principles and focused on students' thinking, with particular emphasis on students' misconceptions, can assist teachers in making more informed instructional decisions. This, in turn, can lead to more effective use of graphics calculators in the classroom and result in significant improvements in student performance, both in operating a graphics calculator and interpreting its output.

The effectiveness of the workshop was evaluated by three means:

- Observations of the teachers during the workshop sessions
- Observations of graphics calculator lessons taught by the teachers after the workshop
- Interviews with their students to assess their graphics calculator proficiency after those lessons.

Participants

Twelve mathematics teachers (2 from each of 6 metropolitan high schools) volunteered to attend a two-day workshop led by the author. The teachers were experienced classroom practitioners with a median teaching time of twenty years and three-quarters of the group having taught for a minimum of fifteen years. They described their teaching styles in traditional terms, and emphasised the need for teachers to demonstrate new concepts to students and allow them plenty of time for practice.

Their knowledge of graphics calculators was extremely limited. Six teachers had not used a graphics calculator prior to the workshop, and the remaining six had participated in one or two brief in-service sessions of the traditional style previously described. Even so, the teachers were interested to learn about graphics calculators and ready to begin using them with their students.

Professional development workshop

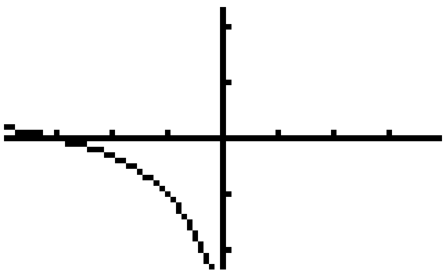
The workshop was held over two consecutive days. The teachers were shown how to operate in the graphing menu of a graphics calculator (the Casio *fx-7400G*) and given information about zooming, tracing and scrolling the graphical images within the viewing window.

The workshop presentations and activities were also designed to inform the teachers about students' thinking and, more specifically, about how students interpret the graphs displayed on the calculator screen. Participants were encouraged to use this knowledge in designing lessons and as the basis for future calculator instruction. The workshop leader did not provide any curriculum materials to the teachers or prescribe any specific teaching approaches. Instead, the teachers were given opportunities to plan lessons for their own classes and present their ideas to the group for feedback.

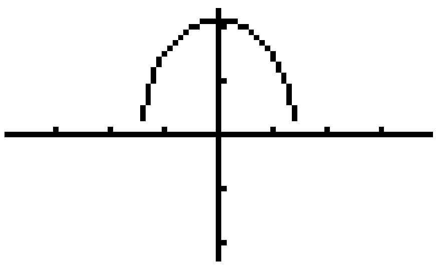
On the first morning of the workshop, the teachers learned the basic operations associated with displaying, investigating and interpreting graphs on the calculator screen. In the afternoon, the teachers worked in groups of four to complete a series of tasks similar to those presented to students in the Phase One interviews, although the teacher tasks were presented at a more sophisticated mathematical level. The purpose of these tasks was to investigate whether the teachers would exhibit any of the errors found previously in the student sample and to engage the teachers in a discussion about how the calculator was operating as it displayed graphs in its viewing window.

Figure 2 shows two of the workshop tasks. The first screen shows the graph of $y = e^x / x$ in the initial viewing window. The function has an unrestricted domain and hence one would expect to see part of the graph in the region of the positive *x*-axis, but the calculator shows only a partial view because of the limited display in the default window. The task investigated whether the teachers would use their knowledge about the symbolic expression of the exponential function to recognise the incomplete graph and what they might do to obtain a more representative image.

The second screen shows the graph of $y = 1.5\sqrt{2 - x^2}$ in the initial window. The function has roots at $x = \pm\sqrt{2}$ and one would expect the graph of the half-ellipse to have *x*-intercepts at those points. However, the values assigned to the pixel columns that comprise the calculator screen in the initial window are rational numbers and there is no pixel value for the irrational roots. Thus the graph appears suspended above the *x*-axis. This task was designed to explore the teachers' knowledge of the underlying processes used by the graphics calculator to assign values to the pixels and highlight them to display a graph.



Sketch a graph of $y = \dfrac{e^x}{x}$ clearly showing any asymptotes or turning points.



Display a graph of $y = 1.5\sqrt{2 - x^2}$.

Is this a reasonable graph of the function?

Can you explain why the graphics calculator displays the graph in this way?

Figure 2. Instructions and initial calculator screens for two of the workshop tasks.

On the second day of the workshop, the central ideas about what students are thinking as they operate a graphics calculator were examined. The researcher presented the results of the Phase One interviews and discussed the students' misconceptions in detail. However, to avoid any possibility of bias in the Phase Two student interview results, the specific interview tasks were not shown to the teachers nor used directly as the basis for this discussion.

The teachers reflected on the Phase One student misconceptions in light of their own experience dealing with similar calculator tasks the previous day. They distinguished between students' mathematical and technical misconceptions and developed categories of errors within each group. The teachers also considered possible reasons for the mathematical and calculator errors commonly made by students.

The workshop leader encouraged the participants to consider how their knowledge of students' misconceptions might be used to re-think some of their instructional practices. Rather than designing instruction to minimise the impact of common student errors, it was proposed that teachers might now choose to use their newly found insights into students' thinking to develop classroom activities that challenged students' conceptual understanding of some key mathematical ideas like scale and approximation. At the same time, knowing more about how students interpreted calculator operations like zooming and tracing functioned, could also assist teachers in designing activities to improve students' basic graphics calculator skills.

The final session of the workshop was given over to lesson planning. The teachers worked in pairs with their school partner to prepare a single lesson that they would be using with their own classes in the coming weeks. The teachers were again encouraged to utilise their knowledge about students' thinking as the basis for the examples, exercises and activities that they devised. The workshop leader suggested that the teachers might include problem solving tasks that would confront some of the likely student misconceptions in their lessons, but he did not specify how the teachers should do this. Each pair presented a brief summary of their lesson to the group and explained how they had developed their ideas. The other participants and the workshop leader gave feedback to the presenters. A total of six lesson plans were examined in this way.

At the conclusion of the workshop, the teachers completed a written evaluation. The sessions in which teachers attempted calculator tasks and when they planned and presented their lesson plans were videotaped for later analysis.

Lesson observations

Six teachers (2 from each of 3 metropolitan high schools) were chosen to take part in the next stage of the workshop evaluation. The choice of teachers was largely determined by the desire to maintain comparability with the Phase One student sample and so teachers were sought from schools with students of similar mathematical ability to those interviewed previously. The researcher observed these teachers using graphics calculators with their Year 10 and Year 11 higher-achieving mathematics classes in the school term immediately following the workshop. Two or three fifty-minute lessons from each teacher were videotaped and lesson notes and worksheets were collected. At the conclusion of each class, the researcher discussed the lesson with the teacher for approximately ten minutes and these post-lesson interviews were also videotaped.

A total of fifteen lessons were observed. Individual teachers taught fourteen of the lessons and one lesson was team-taught by the pair who had attended the workshop from that school. The observations included all of the lessons that these teachers had developed in the lesson planning session at the teachers' workshop. Nine lessons concerned quadratic functions and parabolas, two dealt with linear functions and straight-line graphs, two covered rational functions, and two involved polynomial functions of higher degree.

Student interviews

Fifteen students (5 from each of 3 schools where teachers had been observed) were interviewed at the conclusion of the lesson observations. There were thirteen students in Year 10 (6 male and 7 female) and two in Year 11 (1 male and 1 female) and they were studying mathematics at the highest level available to them. All of the students volunteered to take part in the study at the request of their mathematics teachers. The teachers were asked to choose students with a range of mathematical ability levels from within the observed classes.

Like their counterparts in the earlier study (Mitchelmore & Cavanagh, 2000), these students were pre-calculus and had studied the graphs of straight lines and parabolas. However, they had less experience using graphics calculators than their Phase One counterparts and had only begun using the Casio *fx-7400G* in the lesson observation stage of the study. Typically, the Phase Two students had used graphics calculators for approximately four lessons over two or three week period prior to the first interview. The author interviewed these students using the Phase One protocols.

RESULTS

The workshop sessions

Calculator tasks. All of the teachers exhibited at least some of the errors and misconceptions found in the Phase One student sample. At various times, some teachers failed to recognise an incomplete graph in the initial viewing window, did not take full account of the symbolic representation of a function, were confused when the coordinate axes were unequally scaled, and had difficulty dealing with decimal approximations of irrational numbers. The added cognitive load associated with remembering the correct key-stoke sequences on the calculator and operating in the unfamiliar environment of a relatively small screen of low resolution may have contributed to the prevalence of these mathematical misconceptions among teachers.

The teachers were unable to demonstrate a coherent understanding of the processes used by the graphics calculator to highlight pixels and join them together to create a graph—a result that is neither unexpected nor surprising given that the teachers had only just begun using the technology. It is clear that unless one is shown certain aspects of the calculator's operation, then one is unlikely to develop any real comprehension of how these graphs are produced. This, in turn, leads to technical errors associated with performing basic actions on the calculator. For a complete report of the teachers' performance on the workshop tasks, the reader should consult Cavanagh and Mitchelmore (2003).

The discussions among the teachers as they attempted the workshop tasks anticipated the results of the Phase One interviews that were presented in the subsequent session. In reflecting on their own experiences, the teachers commented on many of the difficulties that they expected their own students would have to deal with as they worked with the graphics calculator. Without prompting from the researcher, the teachers distinguished between technical errors that result from a poor grasp of how the graphics calculator works and conceptual errors associated with certain mathematical ideas. They also identified some of the basic calculator skills and mathematical concepts that would require extra attention in lessons where the technology was present.

Lesson plans. All of the teachers prepared lessons that incorporated problem solving activities for their students, though most could best be described as 'guided discovery' in nature. That is, the teachers carefully structured the lesson activities and prepared worksheets within narrow parameters to provide a scaffold for students' working. At this early stage, the teachers did not yet feel confident enough to allow the students free reign to explore for themselves and so prescriptive exercises were the dominant feature of the lessons.

The lesson plans dealt, in varying degrees, with all of the likely errors and misconceptions that had been discussed at the workshop. The teachers prepared examples of functions that appeared incompletely in the initial viewing window and wanted to encourage students to examine the symbolic representations of these functions closely. The teachers wanted students to form mental images of graphs before they used their calculators. The students would then compare the graphs they expected to see with those displayed on the calculator screen.

Two of the lesson plans encouraged students to experiment with the technology on their own as they explored new concepts, a practice that these teachers did not usually adopt in their classrooms. Both of these lessons focused on changing the coefficients of quadratic functions and examining the transformations in the corresponding parabolas. The teachers intended that students should first be given an opportunity to predict the likely effect of the coefficients on the graphs and then use the calculator to test their conjectures for themselves. The students would then report their findings to the class and the teachers would lead a general class discussion about the results.

Most teachers chose to avoid any consideration of how values are assigned to the pixels by ensuring that the values of all critical points were either integers or simple fractions. However, two teachers did make tentative steps towards including tasks that confronted the difficult question of pixel values, but only in the context of a very teacher-centered approach. The pair proposed to display the graph of a quadratic function with irrational roots on the overhead projector panel and show that it was not possible to locate the exact values of the roots using the graphics calculator. The teachers would position the trace cursor as close as they could to the *x*-axis and the students would watch as the cursor 'jumped' across the axis without indicating the position of the root. A brief discussion about the pixel values would then ensue and the teachers would present a rudimentary explanation of what the calculator had done.

In the process of planning and refining their lesson notes, the teachers became progressively aware of the need to adopt a different pedagogy when using the graphics calculator. They began to recognize the importance of basing instruction on their students' thinking and commented on the need to consider how students would interpret the graphical images displayed on the calculator screen. They carefully selected examples so that students gradually encountered the limitations of the technology and they ensured that fundamental mathematical concepts like scale and approximation were discussed in tandem with the calculator tasks.

The graphics calculator lessons

The style of lessons taught varied in much the same way as the lesson plans presented during the workshop, although there were noticeably fewer discovery learning activities observed. The lessons used the graphics calculator largely as a simple checking device such that students would plot a graph by hand and examine the graph on their calculator to see if what they had drawn was correct. The teacher's main role in these lessons came after the students had finished their work. The teacher then demonstrated examples on the overhead projector panel, highlighted various aspects of the display, and drew students' attention to certain aspects of the calculator's operation.

However, four lessons were structured principally around student investigations of mathematical concepts. In these lessons, the teacher encouraged students to examine the important features of the algebraic forms of various functions and then make predictions about the graphs of these functions before displaying them on the calculator. This approach allowed students to express their conjectures in their own words, strengthened the links between the symbolic and graphical representations of the functions, and alerted students to the dangers sometimes inherent in making judgments about the appearance of a graph based solely on the initial window display.

An important aspect of all of the observed lessons was the way in which the teachers deliberately chose to consider the calculator's technical limitations, and how they did so. All of the teachers drew the attention of students to incomplete and partial graphs, and they discussed approximations in the decimal values of the coordinates displayed on the screen when tracing a graph. The teachers were willing to consider these issues because they better understood the operation of the calculator themselves and because they appreciated the value in exposing their students to these issues as well.

However, the teachers were conscious of the need to sequence examples and exercises carefully so that the limitations of the technology were met gradually. This was evidenced by the fact that whereas in the past the teachers would normally take examples directly from the students' textbooks, they now devised their own. The teachers did so because they were aware that an example that appeared straightforward in a pen-and-paper setting might just as easily become problematic on the calculator screen.

Student interviews

There were many similarities in the student responses in Phase One and Phase Two. All students showed a strong preference for symmetrically scaled coordinate axes and were unable to explain the graphical impact of unequal scales when they encountered them. Both groups of students found it difficult to discern when the decimal coordinates displayed on the calculator screen were rounded values because they were unable to identify when the values of critical points were irrational numbers. And only a small proportion of students in either sample could even begin to describe how values were allocated to the columns of pixels that comprise the calculator screen. These similarities most likely reflect the fact that many of the observed lessons either did not deal with these issues at all, or did so in only a very superficial way.

The Phase Two students did perform significantly better than the Phase One control group in many respects. They were considerably more likely to recognise a partial graph when they saw one displayed in the initial viewing window, and more proficient at zooming out until they displayed a more complete graph. For instance, 67% of Phase Two students solved the first task in Figure 1 compared to only 28% in Phase One.

During the interviews, many of the Phase Two students referred directly to examples and exercises they had completed in class when they were explaining why these graphs did not appear as one might expect in the default window. These students were familiar with such graphs and appeared used to dealing with them.

Phase two students demonstrated a superior understanding of the calculator's zoom function and were better able to predict how the graphical images shown on the calculator screen were transformed under zooming. Here too, there was a strong correlation between students' success in the interview tasks and references they made to similar activities that they had worked through during their lessons.

There was improvement in the students' abilities to deal with the inconsistencies that occur from time to time between the position of the trace cursor and the coordinate values displayed at the bottom of the calculator screen, due to the relatively low resolution of the screen. For instance, it is not uncommon to have the trace cursor positioned exactly on the x-axis, and to see a non-zero y-coordinate shown. So whereas 36% of Phase One students in this situation claimed to have found the exact value of the y-intercept, only 20% of the Phase Two sample made this mistake. This improvement can be reliably attributed to the fact that teachers from the observed classes made a point of drawing their students' attention to this discrepancy and discussing it with them when it occurred.

DISCUSSION

The purpose of this study was to develop and evaluate a teacher workshop focusing on graphics calculators in secondary mathematics and based on the principles of CGI. Clinical interviews were used to gather data about students' thinking as they use a graphics calculator to display and interpret straight line graphs and parabolas. Student misconceptions were identified in two key areas: a lack of understanding of some important mathematical concepts; and a limited understanding of the operation of the technology itself. This information was presented to a group of teachers who were encouraged to use their new-found knowledge of how students interpret the calculator images to prepare and deliver calculator lessons that directly confront likely student errors. It was conjectured that students would improve in both their understanding of the mathematics and in their capacity to use the technology.

The success of the workshop can be judged in terms of three areas: how the teachers responded during the sessions; what they subsequently did in the classroom; and how their students performed on the interview tasks compared to the Phase One control group.

The session in which the teachers attempted calculator tasks similar to those used in the student interviews was extremely valuable. Not only did it expose some of the teachers' own misunderstandings, it also allowed them to gain greater insight into the struggles that their students were likely to have in the classroom. In discussing their own errors and by listening to the workshop leader outline aspects of the calculator's technical operation, the teachers began to recognise that they could not expect to employ graphics calculators effectively in the classroom without re-thinking some of their pedagogical practices.

The report of the Phase One results that immediately followed the calculator tasks also had a much greater impact on the teachers because they could relate the research findings to their own experiences trying to operate the calculator and interpret its graphs. Initially, some teachers were apprehensive about the large number of student errors identified in Phase One and said that they would try to avoid them. However, as the discussion continued, there was a growing realisation among other teachers that not only would it be extremely difficult to avoid dealing with the misconceptions, but that there were benefits to be had from confronting them directly in the classroom.

The workshop participants were experienced classroom practitioners who had developed their personal teaching styles over many years. They described their instructional practices as teacher-centered and commented that their lesson preparation focused primarily on their role as teacher and on what they wanted to teach, so it was quite a change for them to consider students' thinking as the starting-point for devising lesson activities. Even so, they all embraced this new methodology to some extent, at least, and began to consider how the Phase One results could be incorporated into the ways they planned to teach. The lesson plans that they prepared and the comments they made in response to the presentations made by their colleagues clearly demonstrated that all of the teachers were now starting to think carefully about how best to tackle the student misconceptions they were likely to encounter in the classroom.

The lesson observations confirmed that the teachers used their knowledge of students' thinking as the basis for designing lesson activities and materials. All of the lessons included exercises and examples that covered at least some of the misconceptions discussed during the workshop. Some teachers also used problem-solving activities in which students used their calculators to explore graphs of linear and quadratic functions. However, the changes in classroom practice were only incremental. Some teachers still expressed a lack of confidence in their ability to deal with the unusual graphical images often displayed on the calculator and preferred to keep their lessons tightly structured to minimise working with such graphs. The teachers were more comfortable if student explorations were restricted to a small number of examples, carefully prepared and guaranteed to work, rather than a more open-ended approach in which students were free to try new ideas for themselves.

All of the observed lessons included some discussion about how the calculator operates when it reconfigures the viewing window by zooming and scrolling. Issues related to incomplete graphs, unequal scales and decimal approximations were also considered. However, the teacher's confidence in his or her ability to deal with the particular issue was the critical factor in determining how this was done. When teachers felt comfortable with the technology, they allowed students to make predictions about the kinds of graphs that they expected to see and encouraged class discussions designed to resolve any inconsistencies with the calculator's display. In all cases, the complexity of the examples and the sophistication of the class discussion were commensurate with the self-assurance of the teacher.

The areas where the Phase Two student sample performed relatively poorly were those in which the teachers had themselves exhibited misconceptions during the workshop. These included unequal scales, decimal approximations and pixel values. It was the teachers' own lack of ability and confidence in these areas that resulted in more tightly structured lessons that did not adequately deal with the likely student misconceptions. As a result, the students did not significantly improve their understanding of these ideas.

The best results in Phase Two were obtained from students in classes taught where the teachers were confident in their ability to use the calculator and not afraid to allow students to explore new concepts for themselves. These students regularly referred to their classroom experiences when responding to the interview tasks and they often commented that the task they were asked to solve was similar to one they had studied in class.

The results of this study support the view that future graphics calculator inservice programs must do more than simply demonstrate the power of the technology. Teachers need to learn the basic key-strokes required to operate the machine and they must also be given some elementary information about how the calculator functions. In particular, teachers need to know about how the window parameters are set and how the values of the pixels are assigned. They also need instruction about partial views of graphs including the reasons why they are likely to arise and how to use the calculator to display a more complete graph.

This learning requires time and cannot be rushed. A major difficulty with the present study is that the workshop was compressed into two days and many teachers found that there was too little time to absorb all of the information presented to them. If there had been more time available for the workshop, or if the sessions had been spread out over a longer period (say, once a fortnight over a complete school term), then teachers might have developed greater confidence with the calculator. This confidence has already been identified as a key component in achieving better student outcomes.

Teachers also need to know about how students are likely to interpret the graphs they see displayed on the calculator screen and about their errors and misconceptions. Teachers' knowledge of students' thinking is fundamental to the CGI approach and the results of present study confirm the importance of such knowledge.

Teachers need regular and on-going support if they are to integrate new technology effectively. One almost accidental benefit of the teacher workshop relates to the presence of more than one teacher from each school. All of the teachers mentioned this aspect of the workshop in their written evaluations. They all appreciated the support of their partner and noted that it was easier to prepare and present their lessons working together. When they returned to school and began teaching with the graphics calculators, it was also helpful for the teachers to have a colleague to whom they could turn when they needed reassurance or advice.

The role of the workshop leader in any future study should be expanded to include a mentoring role as well. This was one aspect of the present study that differed substantially from the CGI model. In the original CGI study (Carpenter et al., 1989), teachers met regularly to discuss their progress and were assisted as they began to implement change in their classrooms. The present study shows that confidence only builds slowly and long-lasting curriculum reform requires teacher support into the medium term and beyond.

Despite some limitations, the study was largely successful. The results indicate that there is much to be gained by extending the CGI model over a longer time frame. The research findings also support the view that CGI is worth emulating in additional areas of mathematics education and in other knowledge-based domains, such as the sciences, as well.

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