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Graphics Calculators in the Learning of Mathematics: Teacher Understandings and Classroom Practices

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In a two-day workshop, 12 teachers who had not previously used graphics calculators not only learnt how to use a calculator but also completed calculator tasks designed to explore their misconceptions and then learnt about students' difficulties. Observation of lessons taught subsequently by 6 of the teachers, and interviews with 15 of their students, were used to investigate the effectiveness of the workshop in promoting appropriate use of technology in their mathematics classes. It was found that the teachers consistently drew their students' attention to limitations of the technology. Apparently as a result, their students showed considerably fewer misconceptions than students in a previous study who had been taught by teachers who had not undertaken such professional development. The few remaining student misconceptions could be traced to lingering uncertainties on the part of their teachers. Implications of the findings for mathematics teacher education are discussed.

Graphics calculators were first developed in the mid-1980s and since then their use in secondary mathematics classrooms has become widespread. However, most of the research on graphics calculators reported in the literature consists of studies comparing the test results of classes that were taught the same topics using either graphics calculators or a "traditional" approach (Penglase & Arnold, 1996). Only a few studies have attempted to document how use of the calculator helps or hinders student understanding, and we have found no research that specifically examines teachers' understanding of what graphics calculators can and cannot do.

The research described in this paper focuses on educating teachers in the use of graphics calculators in two aspects: (1) Understanding how to use a graphics calculator, including overcoming misconceptions arising from its technical limitations; and (2) learning how to use one effectively in the classroom.

Background

Misconceptions

An important topic in lower secondary school mathematics is the study of graphs of linear and quadratic functions. Students learn to predict the shape of a graph from its equation (straight lines and parabolas) and come to recognise the important features of such graphs (intercepts, the gradient of a line, and the vertex and line of symmetry of a parabola). When drawing these graphs by hand, examples need to be carefully chosen so that distractions are minimised. Examination of textbooks shows that scales are almost always equal, and most

functions have the main features of their graph close to the origin. In addition, the coordinates of critical points are usually integers or simple fractions.

However, the use of a graphics calculator forces students and teachers to confront issues such as unequal scales, partial views, and irrational coordinates. Although the graphics calculator can produce a graph quickly, the individual points are not usually shown and the coordinate axes are not labelled. Thus, there is a real danger that the process of graphing might appear somewhat arbitrary or even magical and that fundamental misconceptions might arise.

Following up on suggestions in several previous studies (Goldenberg, 1988; Mueller & Forster, 1999; Williams, 1993), we recently investigated student misconceptions in detail (Mitchelmore & Cavanagh, 2000). In a series of three clinical interviews, 25 high-achieving Year 10 and 11 students (13 females and 12 males) completed tasks based on linear and quadratic functions that were designed to create situations of cognitive conflict by directly exposing some of the calculator's technological limitations. The interviewer asked the students to explain their thinking as they used the graphics calculator and to interpret its output. Our study confirmed that many student errors in operating a graphics calculator were due to an inadequate understanding of some fundamental mathematical ideas—including scale, accuracy and approximation, and the link between different representations of functions. In particular, students had difficulty interpreting unequal scales and their effects on graphs, interpreting the decimal coordinates displayed when tracing a graph, and recognising when the view window did not display a representative graph of a function. Students also showed a limited understanding of the zoom operation of the calculator and the process used to assign coordinate values to the pixels.

The Teacher's Perspective

The introduction of graphics calculators can provide an impetus for changes in teaching practice, and a small number of studies have investigated whether certain mathematics teaching styles are more compatible with graphics calculator usage than others (Army, 1992; Goos, Galbraith, Renshaw, & Geiger, 2000; Jost, 1992; Rich, 1993). These studies show that “approaches to teaching and learning which emphasise problem solving and exploration, and within which students actively construct and negotiate meaning for the mathematics they encounter, find in this new technology a natural and mathematically powerful partner” (Penglase & Arnold, 1996, p. 85).

Teachers have an important role to play in minimising the impact of student misconceptions (Dick, 1992). In a qualitative case study, Doerr and Zangor (1999) found that the teacher's awareness of the limitations of the technology and her willingness to deal directly with these limitations in classroom instruction were critical factors in teaching students how to apply graphics calculators successfully to their work on graphs of functions. Classroom interactions were characterised by a requirement that students justify the output of the calculator by linking it to the other mathematical concepts they had learned; also, non-calculator strategies were used when appropriate. The students became more sceptical of the calculator's output and learned to check the answers it provided by other means. Drijvers

(2000) came to a similar conclusion in respect of the use of computer algebra systems on a graphics calculator.

However, teachers themselves require education before they can take full advantage of the new opportunities that graphics calculators offer. The traditional approach to professional development in the use of graphics calculators is to instruct teachers in the basic operations of the graphics calculator and provide ready-to-use worksheets and resources for classroom instruction. However, Waits and Demana (2000) argue that this approach is limited and ineffective because it fails to consider many of the broader pedagogical issues associated with the introduction of the technology. In particular, it tends to avoid the technological limitations of graphics calculators and the possibility that teachers themselves might suffer from some of the same misconceptions found in students.

An alternative model for teachers' professional development is that of Cognitively Guided Instruction (CGI). In the CGI model, teachers make instructional decisions based on their knowledge of the development of student's thinking (Carpenter & Fennema, 1992). CGI thus aims to promote the development of teachers' own knowledge of their subject matter while at the same time increasing teacher awareness of the way students learn it. An increasing number of studies have shown the effectiveness of the CGI approach. For example, Tirosh (2000) developed teacher education programs that included a strong focus on familiarising preservice teachers with common thinking processes used by students when operating on fractions. She concluded that informing the teachers about student errors and misconceptions was a significant factor in improving classroom instruction, and recommended that such information should form an essential part of teacher training in this area.

The present study aimed to investigate whether an exploratory professional development program based on the principles of CGI and emphasising student misconceptions about graphics calculators would prove effective in improving teachers' classroom practices and student learning. The professional development was delivered through a two-day teacher workshop and evaluated by three means:

- Observations of teachers during the workshop
- Observations of subsequent lessons in which teachers taught mathematics using a graphics calculator
- Interviews with their students to assess how well they could use the graphics calculator after those lessons

Our basic hypothesis was that informing teachers about the source and nature of students' calculator errors would assist the teachers in making better use of the technology in the classroom and that this, in turn, would lead to significant improvements in students' ability to use a graphics calculator and interpret its output.

Method

Participants

Twelve mathematics teachers (two from each of six Sydney metropolitan high schools) volunteered to attend the workshop. Their teaching experience ranged from one to 29 years, with a median of 20 years. Three-quarters of the participants had taught secondary mathematics for over 15 years.

The participants' knowledge of graphics calculators was very limited. Six of the teachers had not even seen one before attending the workshop. The other six teachers had seen brief demonstrations at in-service courses, but only one had ever used a graphics calculator in the classroom and then only in two lessons. Apart from two teachers who had used *ANUGraph*, a computer graphing software package, the participants had no other experience of using technology in the classroom. However, they all indicated that they were pleased to have an opportunity to increase their knowledge of graphics calculators and agreed to begin using them in their classrooms the following school term.

The Professional Development Workshop

The first author, a practising high school mathematics teacher, designed the materials and led each of the workshop sessions over two consecutive days. The underlying principles guiding the development of the workshop presentations and activities were to inform teachers about students' cognitive processes and misconceptions as they interpreted the graphs shown on a graphics calculator screen, and to provide the participants with an opportunity to learn about effective graphics calculator instruction techniques.

On the first morning of the workshop, the participants learnt how to use the Casio *fx-7400G* graphics calculator. Particular emphasis was given to the basic keystrokes required to display graphs and investigate them by tracing, scrolling the view window, zooming, and re-setting the window parameters.

In the afternoon session, the teachers, working in groups of four, attempted seven graphing tasks designed to confront some of the technical limitations of the machine. These graphing tasks, shown in Figure 1, were similar to those used in the student interviews in the previous study (Mitchelmore & Cavanagh, 2000) but were pitched at a higher level of mathematical sophistication. Tasks 2 and 3 were adapted from questions that appeared in the 1998 Calculus Tertiary Entrance Examination in Western Australia (Curriculum Council, 1998); Task 7 was taken from Day (1993); and the other tasks were specially constructed for this study.

In each task, the teachers displayed the graph in the initial window of the calculator and discussed whether or not the image was a reasonable representation of the function. (The initial window is a default window which, in the case of the Casio *fx-7400G*, has equally scaled coordinate axes marked at unit intervals. In addition, the x -value steps by 0.1 unit as one traces along the graph.) The workshop leader encouraged the participants to explain what they saw and to search for window parameters that would provide a satisfactory graph of the function.

On day two, the workshop leader first presented a session on common student errors and misconceptions and encouraged the teachers to consider how they might deal with them in their lessons. The teachers then worked in pairs (two teachers from the same school) to prepare a lesson incorporating significant use of graphics calculators for one of their classes. Each pair presented a brief summary of

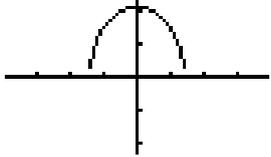
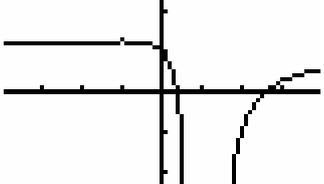
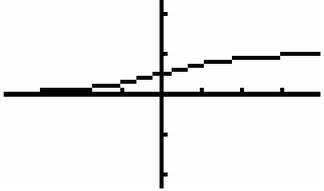
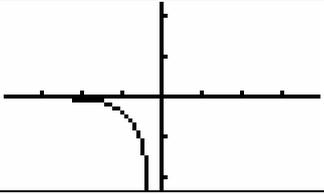
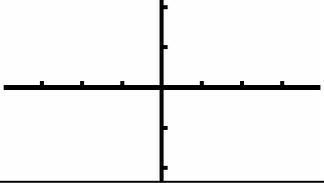
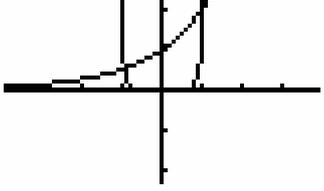
1. Sketch a graph of $y = 1.5\sqrt{2-x^2}$.	
2. Sketch $y = 1 - \frac{x}{(x-1)^2}$ showing any asymptotes or turning points.	
3. Given that $f(x) = (1 + e^{-x})^{-1}$ for $-\infty < x < \infty$, sketch the graph of the inverse of f , $f^{-1}(x)$, clearly indicating all intercepts and asymptotes.	
4. Sketch $y = \sin(60x)$.	
5. Sketch a graph of $y = e^x/x$ clearly showing any asymptotes or turning points.	
6. Sketch $y = \frac{1.04x}{100-x}$ clearly showing any asymptotes or turning points.	
7. How many solutions are there to the equation $2^x = x^{10}$? What are they?	

Figure 1. Teacher workshop graphing tasks and initial graphics calculator screens.

their lesson to the group, and the researcher and the other teachers offered feedback and discussed points arising.

At the conclusion of the workshop, the teachers completed written evaluations of the sessions and materials. The session in which the teachers did the graphing exercises on their calculators, and the session in which they designed and presented their lesson plans, were videotaped and transcribed for detailed analysis.

The Graphics Calculator Lessons

Six teachers (two from each of three participating schools) were chosen to take part in the next stage of the present study. We sought schools that would provide a range of teacher familiarity with graphics calculators, and students of similar mathematical abilities to those interviewed in our earlier study (Mitchelmore & Cavanagh, 2000). The schools were selected on the basis of our observations of the teachers during the workshop and informal discussions with the participants about the nature of their schools and the students they taught. The first author observed the teachers as they taught graphics calculator lessons to either their Year 10 or 11 higher-achieving mathematics classes in the school term immediately following the workshop. Each teacher was observed in two or three 50-minute lessons, all of which were videotaped for later analysis. At the immediate conclusion of each lesson, or as soon as practicable afterwards, the first author discussed the lesson with the teacher for approximately 10 minutes. These brief interviews were also videotaped.

Fifteen lessons were observed in all, 14 of which were taught by individual teachers and one that was team-taught by the two workshop participants from one school. The majority of the lessons (9) concerned quadratic functions and parabolas; 2 lessons dealt with linear functions and straight-line graphs; 2 lessons covered the graphs of rational functions; and 2 lessons considered the graphs of polynomial functions of degree three or four. Worksheets used by the teachers in their lessons were collected.

Student Performance

Fifteen students (five from each of the three schools where teachers had been observed) were interviewed at the end of the teaching period. Thirteen students were in Year 10 (six females and seven males) and two were in Year 11 (one female and one male). All were volunteers who had been requested to participate by their class teachers. We used exactly the same tasks and procedures as in our previous study (Mitchelmore & Cavanagh, 2000).

Results

The Professional Development Workshop

Teacher misconceptions. All of the teachers exhibited one or more of the errors and misconceptions we had previously found in our student sample (Mitchelmore & Cavanagh, 2000). At various times, teachers entered functions into the calculator

incorrectly, failed to recognise incomplete graphs and partial views, could not distinguish between the pixels highlighting the coordinate axes from those used to represent a graph, and did not demonstrate a satisfactory understanding of the processes used by the graphics calculator to display pixels and assign coordinate values to them. The major difficulties on each task were as follows.

In Task 1, the teachers immediately noted that the graph of the half-ellipse did not touch the x-axis as it should, but they could not explain why. They recognised that the intercepts of the half-ellipse were at $x = \pm\sqrt{2}$ and surmised that the irrational nature of the roots may be part of the problem, but no one was able to make any connection with the values assigned to the pixels.

In Tasks 2, 3 and 6 it was not uncommon for teachers to use parentheses incorrectly when entering the rational functions into the calculator. Some teachers had a similar problem in Task 1.

All of the teachers copied the initial graph of Task 4, until one teacher suddenly realised that the display was misleading. (The graph should slope upwards at the origin and have a period of $\pi/30$ radians, not 2.2 or so as it appears to have.) No-one in the group could satisfactorily explain why this graph appeared as it did on the screen. Similarly, in Task 7 all of the teachers assumed that the initial screen showed the complete set of solutions to the equation. Tasks 4 and 7 revealed that many of the teachers had a tendency to accept graphs produced by the calculator without properly relating the images they saw to the symbolic representation of the functions.

In Task 5, all the teachers copied the graph directly from the screen, not recognising that the initial screen of the graphics calculator displayed an unrepresentative graph of the function. One teacher wanted to know why the graph appeared to stop abruptly at $x = -2.2$ when it should clearly have been continuous as the values of x decreased. This teacher, like many others, had not realised that since the values of the function were so small the graph had been superimposed over the pixels that comprised the x-axis. Similar difficulties were experienced in Task 6.

A major source of the above difficulties appears to be the fact that the pixels on a graphics calculator screen can only approximate actual values. This problem also became apparent when teachers manually changed the parameters of the view window and noticed that the tick marks on an axis were sometimes irregularly spaced. So-called "connecting pixels" caused additional difficulties. In the default setting, individual pixels are highlighted to represent calculated points on a graph; then further pixels in the same column are highlighted to connect these and create a smoother curve (see, for example, the initial windows for Tasks 1 and 2 in Figure 1). When tracing a graph, the cursor indicates only the calculated pixels, jumping over the connecting pixels. Some teachers became confused when they noticed that the trace cursor occasionally skipped over some pixels as it moved along the graph.

Lesson plans. In the lessons they developed on the second day of the workshop, the teachers tended to adopt one of two approaches to the use of the graphics calculator. Five of the six lessons followed the same basic sequence: First, the teacher used the overhead projector to demonstrate an example on the graphics calculator. The students observed the teacher and followed each step on their own

graphics calculators. Second, the teacher set a series of exercises on detailed worksheets designed to give the students practice in the concept. Finally, the teacher led a class discussion on each exercise and presented solutions on the overhead projector.

One pair of teachers employed a different strategy. They designed a series of exercises that allowed students to explore a particular concept for themselves with minimal teacher input in the initial stages. They described their plan as “giving [the students] a chance to experiment with [the concept] and discover it for themselves ... There should be time for them to experiment and they can find it out.” When these discovery exercises were completed, the teacher was to encourage the students to discuss their solutions so that the concepts forming the basis of the lesson could be elucidated.

Every lesson addressed at least some of the student errors and misconceptions discussed at the workshop and incorporated examples that dealt with the limitations of the technology. All teachers were careful to refer constantly to the values of the coordinates when tracing as a way of illustrating the behaviour of the function at various points on its graph. They also recognised the possible rounding of the y-coordinates and noted this in their lesson presentations. The lesson plans included graphs that appeared in an incomplete form in the initial window (such as parabolic curves where the vertex is not shown) and points of intersection that could not be seen unless one zoomed out or scrolled. The teachers made it clear during their lesson presentations that such partial views needed to be drawn to the attention of students.

One issue that did not receive much consideration in the lesson plans related to the pixel values. The teachers tended to avoid pixel difficulties by ensuring that all critical points (intercepts, vertices, and intersections) were integer or simple rational values that could easily be found without the need to consider the pixel values in any detail.

The issue of lesson preparation arose during one presentation. As one pair explained to the group:

The graphs we started with were going to be simple graphs of $y = x^2$ and $y = x + 2$. Now if you actually put those on the screen you'll notice that one of the points of intersection actually doesn't exist on the screen. We didn't realise that it would do that! We possibly might choose one that was a bit better than that, but that was the one that we came up with.

The teachers had chosen functions that they assumed would work easily on the graphics calculator to achieve their objective but then discovered some unexpected consequences, as one of the intersection points did not appear in the initial window. The teachers did not wish to confront the issue of partial views so early in the lesson, but they had inadvertently done so. This incident highlighted the need for teachers to choose their examples carefully, and it led to a lively exchange among the group on the importance of thorough lesson planning. It also suggested that the teachers were starting to become aware of some new points related to the effective use of graphics calculators in the classroom.

The Graphics Calculator Lessons

Both approaches adopted by the teachers in the lesson plans they prepared during the workshop were observed in the lessons they subsequently taught. The teachers' confidence in using graphics calculators with students grew slowly as they taught "real" lessons with the graphics calculators, but most still preferred the security of detailed worksheets similar to those that they had presented at the workshop. However, there was one noticeable development in the content of these worksheets compared to those presented at the workshop: The teachers began to incorporate more open-ended tasks and to allow students to make discoveries for themselves using the graphics calculator as an investigative tool.

Three common uses of graphics calculators emerged during the lesson observations.

- A very basic application of the graphics calculator, common to a number of lessons, was the use of the technology as a simple checking device. Students would first produce a graph by plotting points and then display it on the graphics calculator to check whether they had drawn it correctly.
- The graphics calculator was also used as a fast and efficient means of generating examples. This was especially noticeable in some lessons on the basic properties of straight lines and parabolas, when teachers wanted their students to make appropriate connections between the symbolic form of a function and its graph. While teachers ensured that students had sufficient practice in drawing these graphs by hand, they also felt that students' progress could be greatly assisted by examining many examples in order that the links between the various representations of the functions might become clearer.
- A more sophisticated approach observed in some lessons was the use of the graphics calculator as a device to encourage and improve predictions about the graphs of functions without plotting. Here the teacher would encourage students to make predictions about the important graphical features of the functions that they were investigating before displaying them on the calculator screen. This approach proved extremely valuable in focusing students' attention on the symbolic representations of the functions, thereby strengthening the links between the algebraic and graphical forms and alerting students to the danger in uncritically copying the first graph that they saw on the screen.

An important aim of the lesson observations was to assess the degree to which the teachers considered the limitations of the graphics calculator, and the ways in which they did this. All of the teachers made a conscientious effort to draw their students' attention to the coordinate values when tracing, particularly the value of the y-coordinate. The teachers also encouraged students to make predictions about the kinds of coordinates they would expect to see at intercepts, for example, and to justify their solutions by explaining how the coordinates they saw on the graphics calculator screen supported these predicted values.

Teachers also tried to make students aware of the possible rounding of the y-coordinates displayed when tracing. Most of the examples and exercises observed during the lessons were relatively simple functions that were examined in the initial window where rounding was not a common occurrence. However, when the window parameters were manually reset or modified after repeated zooming, rounding of the y-coordinates commonly occurred.

Every teacher dealt with incomplete graphs and partial views. Examples covered in the lessons included critical points such as points of intersection, vertices, and intercepts which did not appear in the initial view window, and graphs whose basic shape was not completely seen at first. However, many of the examples that the teachers chose were relatively trivial in nature, and it was quite easy for students to recognise that they only saw a partial view of the function and scroll the view window once or twice to reveal a complete graph. A more

challenging example was the graph of $y = \frac{x+3}{x-1}$, shown in Figure 2. Here an entire

section of the graph on one side of the asymptote does not appear on the initial screen. In cases such as this, the students were more likely to copy the initial image directly from the calculator screen. So, examples of this kind afforded teachers an excellent opportunity of bringing such partial views to the students' attention and led to profitable discussions on this important characteristic of the calculator screen. Such examples also assisted teachers in emphasising the importance of what might be termed algebraic estimation, whereby one tries to imagine the likely shape of a graph based on an inspection of its algebraic form.

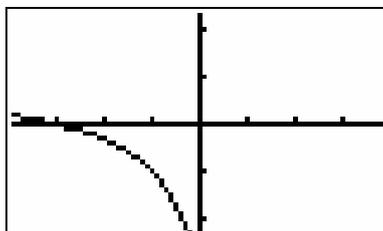


Figure 2. The graph of $y = \frac{x+3}{x-1}$ in the initial window.

Few teachers attempted to deal with issues concerning the values assigned to the pixels on the graphics calculator screen. But some instances did occur where questions about the values assigned to the pixels arose in spite of the best intentions of the teacher to avoid them. This situation occurred most often when the value of a critical point lay between the values assigned to the adjacent pixel columns. For example, one teacher used the overhead projector screen to display a graph of $y = x^2$ in the initial window so that she could show her students how to locate the point where $y = 5$. As the teacher traced along the curve, the cursor jumped from (2.2, 4.84) to (2.3, 5.29) and some students asked her why the point at

$y = 5$ could not be seen. The teacher responded by saying, "At the moment, it's not giving me the exact answer and we may not even get an exact answer, but perhaps if we zoom in we might be able to get a value for it". She then zoomed in and re-traced the curve but could not display the point. In the interview immediately following the lesson, the teacher recognised that the solution to $x^2 = 5$ was irrational, but she had difficulty seeing that such values could not be displayed exactly on the graphics calculator. The teacher's suggestion to her class that they might eventually be able to display the point by zooming is a commonly held misconception, indicating lack of understanding of both the zoom operation and the process used to assign pixel values.

All teachers placed a great deal of importance on using equally scaled axes and regarded them as the most appropriate environment in which to view and explore a graph. They rarely used unequal scales in their lessons, preferring instead to operate almost exclusively within the initial window. Even when it was necessary to change windows, the teachers always did so using either the zoom-in function or scrolling (operations which maintain the equal scaling). The teachers were aware of the zoom box operator and the possibility of setting the window parameters by hand, but they never used it. Moreover, when the students themselves occasionally accessed the zoom box facility and inevitably produced graphs where the axes were not equally scaled, the teachers would generally direct them to return to the initial window and repeat the zoom without using the box option.

The teachers rarely used questions taken from textbooks, preferring instead to choose examples of their own. One teacher stressed this point in a discussion after her lesson:

I usually use the textbook a lot more, but I wanted to choose the examples myself. I wanted graphs that showed up some of the limits of the calculator more ... I think [the students] need to see that the calculator isn't perfect, that it can't always give them the full picture straight away. Now that I know what some of the limitations are myself, and I know why they happen, I'm more confident to show them.

Student Performance

Like their counterparts in our earlier study (Mitchelmore & Cavanagh, 2000), the students in this study had studied graphs of straight lines and parabolas. They were also familiar with the quadratic formula, and had used it to solve quadratic equations and locate the vertex and x-intercepts of a parabolic graph. However, the students in the present study had less experience with graphics calculators, having only begun using the Casio fx-7400G at the beginning of that term. Typically, they had worked with graphics calculators for approximately 4 lessons over a period of 2 to 3 weeks prior to their first interview. Nevertheless, there were many similarities between the responses of the students in the two studies. In both cases, students showed a strong preference for equal scales and experienced difficulties whenever they were asked to interpret graphs with unequally scaled axes. Also, use of the graphics calculator revealed a fundamental lack of understanding of rationality on the part of the majority of students. For example, very few students

were able to identify whether a critical point on a graph did or did not have a rational x-coordinate.

All teachers used the zoom facility of the calculator in their lessons, but few discussed precisely how zooming affects the window display. It is not surprising, therefore, that few students could explain the calculator's zoom operation and its impact on a graph. All students spoke of zooming-in as one might describe the use of a magnifying glass, whereby hitherto unseen details of an object are gradually revealed. They were unable to link the operation of zooming with any change in the scale of the graphs displayed in subsequent viewing windows. Only 13% (2 out of 15) students in the present study could describe how the increment between adjacent columns was calculated or demonstrate any understanding of how the value of the increment was affected by zooming. This figure is not very different from the low 8% (2 out of 25) found in the earlier study (Mitchelmore & Cavanagh, 2000).

However, one teacher who did refer to the underlying processes of the zoom function of the graphics calculator during his lessons was quite successful in conveying this aspect of the calculator's operation to his students. The students who were interviewed from this class showed a considerably better understanding of what the calculator was doing when they zoomed in and were able both to anticipate and explain the effects of zooming. For instance, they all correctly predicted that a horizontal line of pixels near the vertex of a parabola would become longer after each zoom operation. They were also able to explain how the scales on the axes changed by referring to the reduction in the increment between adjacent columns of pixels.

There were some more widespread and substantial improvements made by the students in the present study. So whereas only 28% of students in the previous study (Mitchelmore & Cavanagh, 2000) recognised a partial view of a parabola in the initial window, 67% of students in the present study readily zoomed out to obtain a representative graph of this quadratic function. As one student remarked:

I'm only seeing part of the graph because the screen is very limited and, as we did in class, you have to move around until you get it. It depends on what you're kind of looking for. You could be looking for where its intercepts are, or you could be looking for what's the minimum point, or maybe all you want to see is the shape of it.

The reference to work she had previously done in class confirms that the students' learning was directly related to her teacher's practice in drawing students' attention to the possibility of being misled by the appearance of the graph in the initial window.

Students in the present study made significantly fewer errors related to misinterpreting or disregarding the y-coordinate. For instance, 36% of students in the earlier study incorrectly claimed that they had located the x-intercept of a parabola when they positioned the cursor on the x-axis, even though the y-coordinate was displayed as -0.006. Only 20% of the students in the present study made this error. The improvement can be reliably attributed to the fact that the teachers regularly asked students to interpret the displayed coordinates of critical points on a graph.

Students in the present study were also more adept at recognising that the y-coordinates were calculated values. Their comments showed that they understood how these values were obtained by substituting the corresponding x-coordinates into the algebraic form of the function. The students also explained that these calculated y-values were often rounded because the Casio *fx-7400G* only displayed a relatively small number of significant figures on its graphing screen. Teachers regularly drew attention to these two important characteristics of the y-coordinates, and this practice assisted many students to interpret the numerical information supplied by the calculator correctly.

Discussion

Even though the evaluation of the professional development workshop was limited in size and scope, nonetheless certain results are clear. After learning about misconceptions that can arise when using graphics calculators, the teachers did emphasise the limitations of the technology in their lessons and led students to directly confront apparent inconsistencies. As a result, the students in the present study quickly became competent at using the output of the calculator to solve mathematical tasks, their performance comparing favourably with that of an earlier sample whose teachers had not learned about student misconceptions (Mitchelmore & Cavanagh, 2000). The results of the present study thus provide clear evidence to support the view that informing teachers about likely student misconceptions can greatly assist them in lesson preparation and classroom instruction and may lead to significant gains in their students' ability to use the technology effectively.

The results of the present study also support the findings of Steele (1994) that teachers must do more than simply show students examples of misleading graphs on the graphics calculator. It is also necessary to reinforce the links among the different representations of the function (Dugdale, 1993; Leinhardt, Zaslavsky, & Stein, 1990; Moschkovich, Schoenfeld & Arcavi, 1993) and continually draw students' attention to any discrepancies between the kind of graph they expect to see and the image that the graphics calculator produces.

The traditional model of teacher professional development in the use of graphics calculators, so roundly criticised by Waits and Demana (2000), is ineffective because it focuses almost exclusively on the basic operation of the machine. The calculator examples teachers see in such training sessions often avoid many of the issues raised in this study (partial views, unfriendly window settings, pixel-related problems, and so on). As a result, we believe that teachers are not supplied with all the knowledge and skills they need to make the best use of graphics calculators in the classroom. Moreover, if teachers move beyond the worksheet examples they have been given and attempt to display graphs of other functions, they will encounter situations they will be unable to explain and become instantly discouraged from using graphics calculators more widely.

Even if the training program used in this study was effective, there is still room for improvement. It is clear that teachers need more explicit instruction on the processes the calculator uses to highlight pixels, represent them by coordinate values, and display graphs (Dowsey & Tynan, 1998; Goldenberg, 1988). Armed

with this knowledge, teachers would have a more complete understanding of how the graphics calculator operates and be able to confront unexpected graphical displays more effectively in the classroom.

One almost accidental beneficial feature of the training program was the presence of more than one teacher from each school. The teachers generally felt more comfortable attending the workshop with a colleague and appreciated the fact that when they returned to their schools there would be another person on the staff with whom they could consult if problems arose. Some teachers also took the opportunity to begin using the graphics calculators by team teaching and reported that this made it easier for them to take the first steps in using the calculators with students.

As our study has made clear, teachers' confidence in their own understanding of the calculator's operation is crucial in determining how effectively they will use the technology in the classroom. Teachers who felt unconfident tended to design lessons that were tightly structured and securely teacher-centred. But confidence only builds slowly, and considerable experience using graphics calculators in the classroom is needed before teachers can become sufficiently confident for lasting change to occur. Future professional development programs will thus need to ensure that more on-going support for teachers is provided, not only when teachers first begin to initiate changes, but over the medium to long term as well. One way of doing this would be to spread out the professional development program over a longer period (e.g., with teachers meeting for two or three hours each fortnight over a school term). In this way, each session could be more narrowly focussed, so that teachers are not overwhelmed with too much detail all at once, and teachers would have a chance to experiment a little at a time in the classroom, gaining feedback from the instructor and their peers at regular intervals. Similar conclusions apply to preservice teacher education.

An important issue that arose in this study is when to avoid technical limitations and when to confront them more directly. This aspect of instruction was not adequately addressed during the workshop and the teachers needed more guidance and support in this regard. Care certainly needs to be taken in the selection of teaching examples, and there is a strong case for structuring exercises so that difficulties are minimised in the early stages. However, unless more conceptually demanding examples that push the machine's technical limits are eventually included, students will probably not make sufficient progress in their understanding. The evidence of this study is that, when teachers force students to confront possible misconceptions, not only is students' understanding of the technology strengthened (Dick, 1992; Kissane & Kemp, 1999) but they also learn more mathematics.

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