Secrets of the crystal ball

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A fascinating email that has been doing the rounds alerts the recipient to a crystal ball known as "The Flash Mind Reader" (Naughton, 2006). It claims to have magical powers and can be found at the URL:
http://albinoblacksheep.com/flash/mind.php

In effect, it is essentially a mathematics game in which the player is invited to select any two-digit number (where the leading digit may be zero) and then subtract the sum of these two digits from the original number. A chart is provided (such as that in Table 1) in which the (adjusted) number they obtained will have a symbol next to it.

For example, if the number 65 is selected, the player would subtract 11 to obtain 54. From Table 1, the symbol for 54 is a Ω. They are then asked to click on the crystal ball displayed on the screen near the chart. When they do so, the correct symbol (of over a dozen possible symbols in the chart) mysteriously appears. The player is invited to try again and even if they select a different number, and obtain a different symbol, the crystal ball always seems to be correct. This can be quite unnerving but a little mathematics can reveal the secret of its success.

Two-digit numbers

Suppose the player selects the number A_2A_1 consisting of the two digits A_2 and A_1. The actual number is $10A_2 + A_1$. The sum of the digits is $(A_2 + A_1)$, which when subtracted from A_2A_1 yields the number $9A_2$. That is, it will always be a multiple of 9. The only possibilities for the answer obtained by the player are therefore 0, 9, 18, 27, 36, 45, 54, 63, 72, 81 and will naturally vary according to the original number selected.

The supplied chart therefore places the same symbol next to these ten numbers while the remaining ninety numbers can have any other symbol since they will never arise. In other words, they represent 90 per cent of the supposedly feasible answers but are merely masking symbols.

An example is given in Table 1 where the symbol Ω is placed next to these

Table 1. Typical symbol table for a two-digit number game.

Adjusted No.	***Symbol***	***Adjusted No.***	***Symbol***	***Adjusted No.***	***Symbol***	***Adjusted No.***	***Symbol***
00	Ω	25	ξ	50	#	75	Ω
01	&	26	*	51	∂	76	⊗
02	∂	27	Ω	52	⊗	77	∂
03	€	28	∂	53	*	78	#
04	⊗	29	€	54	Ω	79	ξ
05	!	30	⊗	55	€	80	*
06	ξ	31	Ω	56	!	81	Ω
07	€	32	#	57	∂	82	&
08	Φ	33	∂	58	Φ	83	*
09	Ω	34	ξ	59	⊗	84	∂
10	*	35	!	60	€	85	∂
11	&	36	Ω	61	ξ	86	⊗
12	€	37	ξ	62	!	87	#
13	Ω	38	*	63	Ω	88	€
14	#	39	Φ	64	!	89	Φ
15	ξ	40	€	65	#	90	⊗
16	&	41	∂	66	!	91	&
17	∂	42	!	67	Φ	92	ξ
18	Ω	43	⊗	68	⊗	93	*
19	*	44	#	69	&	94	#
20	&	45	Ω	70	*	95	⊗
21	⊗	46	!	71	&	96	&
22	ξ	47	*	72	Ω	97	∂
23	Ω	48	ξ	73	Φ	98	€
24	#	49	&	74	€	99	ξ

only ten possible numbers. The remaining ninety numbers have an assortment of other symbols. Whatever the two-digit number the player chooses, the symbol Ω will be the one that the crystal ball guesses will be their answer.

When the game is played again, the table re-appears but with a different arrangement of the symbols, although the symbol corresponding to the ten possible outcomes will be the same one, although not the same symbol assigned to them in the previous game. However, this is the symbol that the crystal ball will guess and it will always be correct. Indeed, the player usually does not notice that the table itself has changed and is amazed that the crystal ball is able to determine the correct symbol every time.

Three-digit numbers

The theory can easily extend to three-digit numbers where the player selects any three-digit number and subtracts the sum of the three digits from it. Suppose the player makes a selection of the three-digit number $A_3A_2A_1$. When $(A_3 + A_2 + A_1)$ is subtracted from this number the answer will be $99A_3 + 9A_2$. Since A_3 and A_2 must be in the range 0–9, there are only 100 possibilities for the answers. These are members of the sequence 0, 9, 18, … 972 (with the exclusion of the nine numbers 90, 189, 288, 387, 486, 585, 684, 783, 882 that cannot be obtained).

Once again only 10% of the possible 1000 seemingly feasible answers can actually be obtained by the player and so 900 "red herring" symbols could be placed in the crystal ball chart and the main symbol (the one that the crystal ball guesses) placed next to the 100 possibilities.

The general problem

The crystal ball can easily extend itself to a situation with any number of digits. Suppose that the player is invited to select any n-digit number and chooses $A_nA_{n-1}A_{n-2}\ldots A_2A_1$ where the A_i (i = 1 to n) can be anywhere in the range 0–9. When the sum of the digits $(A_n + A_{n-1} + A_{n-2} + \ldots + A_2 + A_1)$ is subtracted, the resulting the number the player obtains is shown in (1).

Resulting number for the player

$$= (10^{n-1} - 1)\mathrm{A}_n + (10^{n-2} - 1)\mathrm{A}_{n-1} + \ldots + (10^1 - 1)A_2 \qquad (1)$$

Since each of the numbers A_n, A_{n-1}, … A_2 are in the range 0–9, there are 10^{n-1} possible answers that the player could obtain. However, there are 10^n n-digit numbers (allowing leading zeros) and so once again there will always be 90% of the n-digit numbers that cannot be a possible answer. For example, for a five-digit number selected (where n = 5), there are only 1000 possible (adjusted) numbers and 9000 (adjusted) numbers than can never be obtained.

Although the crystal ball seems to have all the answers, and manages to frighten some of the unsuspecting players, it is a wonderful example of how mathematics can be used in an absorbing way and provides an interesting challenge for students to uncover the secret behind its mysterious powers.

Reference

Naughton, A. (2006). *The Flash Mind Reader.* Accessed at http://albinoblacksheep.com/flash/mind.php