Bitcoin Option Pricing With a SETAR-GARCH Model

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ABSTRACT
This paper aims to study the pricing of Bitcoin options with a view to incorporating both conditional heteroscedasticity and regime switching in Bitcoin returns. Specifically, a nonlinear time series model combining both the self-exciting threshold autoregressive (SETAR) model and the generalized autoregressive conditional heteroscedastic (GARCH) model is adopted for modelling Bitcoin return dynamics. Specifically, the SETAR model is used to model regime switching and the Heston-Nandi GARCH model is adopted to model conditional heteroscedasticity. Both the conditional Esscher transform and the variance-dependent pricing kernel are used to specify a pricing kernel. Numerical studies on the Bitcoin option prices using real bitcoins data are presented.

Keywords: Bitcoins options; regime switching; conditional heteroscedasticity; conditional Esscher transform; variance-dependent pricing kernel; threshold autoregressive models

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1. Introduction

Bitcoins and blockchain technologies appear to be major innovations in the field of FinTech. The origin of these innovations may be traced back to Sato Nakamoto and his group (Nakamoto (2009)). Since its inception in 2009, there has been a tremendous growth in the trading activities of Bitcoins and other cryptocurrencies around the globe. According to CoinMarketCap, as of January 2020, the market capitalization of Bitcoin is $147,980,187,842 USD. Bitcoins have been considered by some authors as

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investment assets. See, for example, Bell (2013), Yermack (2013), Böhme et al. (2015), Baur et al. (2015), Glaser et al. (2014) and Baur and Dimp (2017). Specifically, it was concluded in Yermack (2013) that Bitcoins are to be thought of as an asset or a speculative investment. See also Gronwald (2014). Trading Bitcoins may be thought of as risky due to their highly speculative and volatile nature, (see, for example, Grinberg (2011), Cheah and Fry (2015) and Katsiampa (2017)). Yermack (2013) found that Bitcoin prices are more volatile than prices of commodities such as gold. It was found in Osterrieder and Lorenz (2017) that compared with the volatility of G10 currencies, the volatility of Bitcoins could be six to seven times higher. Furthermore, there were reversals in trading activities and market values of Bitcoins at the beginning of 2018, which may be attributed to the tightening of regulations for Bitcoin trading in some jurisdictions and pessimistic market expectations, (see, for example, Russo and Lam (2018)). It appears that changes in market regimes may have a significant effect on the market values of Bitcoins as noted in Siu (2019a). Consequently, how to hedge and manage the risk from Bitcoin trading may represent a practically relevant and important issue.

Derivatives provide a practical means to hedge and manage the risk from trading financial securities or assets. It seems natural to consider the possibility of using derivatives to hedge and manage risk from trading Bitcoins and other cryptocurrencies. A call for the development of derivatives markets for Bitcoins and other cryptocurrencies seems to be relatively recent. Indeed, it was noted in Böhme et al. (2015), page 220, that up to 2015, derivatives markets in Bitcoin were rare. Given the practical demand for hedging and managing the high volatility risk of Bitcoin, on 18^{th} December 2017, the Chicago Mercentile Exchange (CME) launched the trading of Bitcoin futures, namely BTC futures, whose underlying is the CME CF Bitcoin Reference Rate (BRR), (see Hou et al. (2019)). There have been some discussions concerning the possibilities of trading Bitcoin options. See, for example, the article titled "Bitcoin Options Are Headed to The U.S." by Hankin (2017) in INVESTOPEDIA. It appears, however, that the development of Bitcoin derivatives markets may still be at its infancy relative to the mature markets for derivatives written on other financial securities. One of the important issues for the development of Bitcoin derivatives markets is to address the question on how Bitcoin derivatives may be priced. Option pricing theory and technologies may provide a scientific way to address this question. Recently, Hou et al. (2019) developed a model for pricing cryptocurrency options, particularly options on the cryptocurrency index (CRIX), based on an affine stochastic volatility with correlated jumps model coupled with Bayesian Markov Chain Monte Carlo simulation. Zaitsev (2019) estimated the empirical distribution for future returns from Bitcoins using Bitcoin option prices by applying the Breeden-Litzenberger state-contingent prices approach (Breeden and Litzenberger (1978)), where the theoretical basis of the latter rests on the no-arbitrage principle, the risk-neutral valuation and the Arrow-Debreu state-price securities. Pagnottoni (2019) adopted a neural network approach for pricing Bitcoin options. Alexander et al. (2020) studied the price discovery and informational efficiency on BitMEX bitcoin derivatives. Shi and Shi (2019) studied Bitcoin futures markets. There were also some works on the price discovery and valuation in Bitcoin or Cryptocurrency markets. See, for example, Momtaz (2019) and Makarov and Schoar (2019b).

(Nonlinear) time series models may be applied to describe the dynamical behavior of Bitcoin returns. Some advocates of time series approach to Bitcoin returns, for example, Ciaian et al. (2016) and Bariviera et al. (2017), seem to hold the view that Bitcoin returns may not be driven by macro-financial indicators. The generalized autoregressive conditional heteroscedastic model and some related models were adopted to model an important feature of Bitcoin returns, namely conditional heteroscedasticity, in, for example, Bouoiyour and Selmi (2016), Dyhrberg (2016), Katsiampa (2017), Bouri et al. (2017), Stavroyiannis (2017), Colucci (2018) and Siu (2019a, b). Gronwald (2014) pointed out that a major concern for the empirical features of Bitcoin prices is the volatility observed in the Bitcoin market and adopted an autoregressive jump-intensity GARCH model to analyse Bitcoin prices empirically. Li and Wang (2017) studied market values of cryptocurrencies using an autoregressive distributed lag model coupled with a bound test approach. Lehner et al. (2018) adopted the GARCH model for studying cryptocurrency valuation models. Siu (2019a) adopted both the self-exciting threshold autoregressive model and the generalized autoregressive conditional heteroscedastic model to describe Bitcoin returns as examples to illustrate the applications of Bayesian nonlinear expectations. Magtanggol De Guzman and So (2018) provided an empirical analysis for Bitcoin prices using a threshold-in-mean model in the presence of both exogenous variables and GARCH innovations. Kristjanpoller and Minutolo (2018) predicted the price volatility of Bitcoin using a hybrid model which combined the GARCH model, the artificial neural network and the principal components analysis. Caporale and Zekokh (2019) adopted Markov-switching GARCH models to modelling volatility of cryptocurrencies.

This paper aims to investigate the pricing of Bitcoin options with a view to capturing both conditional heteroscedasticity and regime switching in Bitcoin returns. A nonlinear time series model combining both the self-exciting threshold autoregressive (SETAR) model and the generalized autoregressive conditional heteroscedastic (GARCH) model, say a SETAR-GARCH model, is adopted for Bitcoin return dynamics. Specifically, the SETAR model is used to model regime switching and the Heston-Nandi GARCH model in Heston and Nandi (2000) is adopted to describe conditional heteroscedasticity. A closely related model, namely the SETAR-ARCH model, was proposed in Tong (1990), Chapter 3, and it was developed there as a second-generation nonlinear time series model. The SETAR-ARCH model was employed for pricing options in Siu et al. (2006). Here the pricing of Bitcoin options is done by following the pricing methodology in Siu et al. (2006), which is related to the methodology based on the conditional Esscher transform for GARCH models in, for example, Siu et al. (2004), Elliott et al. (2006), Badescu and Kulperger (2008) and Christoffersen et al. (2010). We shall also adopt the variance-dependent pricing kernel to price Bitcoin options under a GARCH model [1]. The variance-dependent pricing kernel incorporates the variance component in the specification for a pricing kernel and was considered in the literature. See, for example, Corsi et al. (2013), Christoffersen et al. (2013), Christoffersen et al. (2015), Majewski et al. (2015), Bormetti et al. (2016), Badescu et al. (2017, 2019), Bormetti et al. (2019), Alitab et al. (2020). As in Christoffersen et al. (2013), we use the Heston-Nandi GARCH model here when discussing the use of the variance-dependent pricing kernel. One advantage of using the Heston-Nandi GARCH model is that the GARCH structure is preserved under the risk-neutralization by the variance-dependent pricing kernel from which the unconditional risk-neutral variances can be obtained analytically (Christoffersen et al. (2013)). Another advantage of using the Heston-Nandi GARCH model is that the option pricing formula based on the characteristic function of the terminal underlying price in Heston and Nandi (2000) is applicable in the case where there is a variance-

[1] We would like to thank the comments from a referee which inspired us to consider a variance-dependent pricing kernel in this paper.

dependent pricing kernel. The option pricing formula by Heston and Nandi (2000) can be implemented using numerical integration. Indeed, Christoffersen et al. (2013) adopted the option pricing formula by Heston and Nandi (2000) for the case of the variance-dependent pricing kernel. One may consider the use of an option pricing model under a Markov-switching Heston-Nandi GARCH model (see, for example, Elliott et al. (2006)), to incorporate conditional heteroscedasticity and regime switching in conditional volatility in valuing Bitcoin options. However, the focus of this paper is the SETAR-GARCH model. As noted in, for example, Siu (2016), an advantage of using a SETAR model is that regime switches may be incorporated without introducing an underlying modulating process, say a Markov chain. Furthermore, in a recent paper by Chevallier et al. (2019), a Markov-switching Lévy jump-diffusion model was considered for studying the price dynamics of Bitcoin. Again, the model structure and regime-switching mechanism in Chevallier et al. (2019) are different from those considered here. Specifically, their regime-switching mechanism was governed by a continuous-time finite-state Markov chain while the regime-switching mechanism considered here is described by the self-exciting threshold principle. Garnier and Soln (2019) considered the use of the power-law parameters for the identification of regime switches in the Bitcoin price. Again their approach appears to be different from those considered in this paper. Numerical studies on the Bitcoin option prices using real Bitcoin exchange rate data are presented. Bitcoin option prices under the SETAR-GARCH model with the Heston-Nandi GARCH component using the conditional Esscher transform are computed by the Monte Carlo simulation. Whereas, Bitcoin option prices under the Heston-Nandi GARCH model using the variance-dependent pricing kernel and the conditional Esscher transform are computed by the option pricing formula in Heston and Nandi (2000) using numerical integration. Comparisons among Bitcoin option prices matrices with different strikes and maturities from the SETAR-GARCH model with the Heston-Nandi GARCH component, against those from the Heston-Nandi GARCH model and the standard Black-Scholes-Merton model are provided. The behavior of the Black-Scholes implied volatilities for European Bitcoin call options from the SETAR-GARCH model with the Heston-Nandi GARCH component and the Heston-Nandi GARCH model is also studied. Hou et al. (2019) provided empirical studies on the CRIX using certain GARCH-type models such as the t-GARCH, EGARCH, ARIMA-t-GARCH and LGARCH models. However, it does not seem that they focused on the use of GARCH-type models for pricing cryptocurrency options. Furthermore, they did not consider the regime-switching effect described by the SETAR model. We shall also provide numerical studies for the impacts of incorporating the variance component in the specification for a pricing kernel under the Heston-Nandi GARCH model on the Bitcoin option prices and their implied volatilities. This paper does not intend to discuss whether Bitcoin derivatives should be traded or not. It intends to provide some insights into understanding consequences for Bitcoin option prices of empirical features such as conditional heteroscedasticity due to the GARCH effect, regime switches due to the SETAR effect and their combination.

The next section presents the model and the pricing methodology. The numerical studies and results are presented in Section 3. Some concluding remarks and potential topics for further research are provided in the final section. Tables and figures presenting the numerical results are presented after an Appendix.

2. The Model and Methodology

We begin with a discrete-time economy, where the time parameter set of the economy is $\mathcal{T} := \{0, 1, 2, \cdots\}$. Whenever necessary, we may sometimes consider the time parameter set $\mathcal{T}_1 := \{1, 2, \cdots\}$ starting from time one. A complete filtered probability space $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$ is adopted to describe market uncertainty and the evolution of market information over time, where $\mathbb{F} := \{\mathcal{F}_t\}_{t\in\mathcal{T}}$ is a filtration and $\mathbb{P}$ is a real-world probability measure. Here, for each $t \in \mathcal{T}$, the information set $\mathcal{F}_t$ up to and including time t is supposed to be the σ-field $\sigma\{S_u|u \in [0, t]\}$ generated by $\{S_u|u \in [0, t]\}$, where $\{S_t\}_{t\in\mathcal{T}}$ is the process of the Bitcoin exchange rate and S_t represents the value of one unit of the Bitcoin in terms of a currency, say the US dollar. For each $t \in \mathcal{T}_1$, let R_t be the logarithmic return from the Bitcoin exchange rate during the time period from time $t - 1$ to time t, (i.e., the t^{th} period), say $R_t := \ln(\frac{S_t}{S_{t-1}})$. For each $t \in \mathcal{T}_1$, let h_t denote the conditional variance of the logarithmic return R_t given $\mathcal{F}_{t-1}$. That is, $h_t = \mathrm{V}ar[R_t|\mathcal{F}_{t-1}]$.

Let $\{\epsilon_t\}_{t\in\mathcal{T}_1}$ be a sequence of independent and identically distributed (i.i.d.) standard normal random variables under $\mathbb{P}$. That is, for each $t \in \mathcal{T}_1$, $\epsilon_t \overset{\text{i.i.d.}}{\sim} N(0, 1)$, under $\mathbb{P}$. Then we assume that under $\mathbb{P}$, the logarithmic returns $\{R_t\}_{t\in\mathcal{T}_1}$ from the Bitcoin exchange rates and their conditional variances $\{h_t\}_{t\in\mathcal{T}_1}$ are governed by the simple SETAR-GARCH model with the Heston-Nandi GARCH (1,1) component in Eq. (2.1) and Eq. (2.2) below:

$$R_t = \begin{cases} r_d - r_s + \left(\lambda_1 - \frac{1}{2}\right)h_t + \sqrt{h_t}\epsilon_t & \text{if } R_{t-1} \geq r; \\ r_d - r_s + \left(\lambda_2 - \frac{1}{2}\right)h_t + \sqrt{h_t}\epsilon_t & \text{if } R_{t-1} < r, \end{cases} \tag{2.1}$$

and

$$h_t = \alpha_0 + \alpha_1(\epsilon_{t-1} - \gamma\sqrt{h_{t-1}})^2 + \beta h_{t-1}. \tag{2.2}$$

Throughout this paper, when we refer to the SETAR-GARCH model or the SETAR-Heston-Nandi-GARCH model, we mean the one with the Heston-Nandi GARCH(1, 1) component. Here we consider a modified version of the Heston-Nandi GARCH(1,1) model in Christoffersen et al. (2013), Equation (5) therein. Specifically, in the modified version of the Heston-Nandi GARCH(1,1) model, the risk-free rate of interest is given by $r_d - r_s$ and the risk premium in the Bitcoin market can switch over time between two values λ_1 and λ_2 in accordance with the threshold principle of the SETAR model; r_d is the interest rate from the U.S. Treasury bills while r_s is the short-term interest rate from the Bitcoin (see, for example, Posedel (2006) in specifying the conditional mean in a GARCH model for foreign exchange returns); λ_1 and λ_2 are the risk premiums in the "Good" and "Bad" regimes of the Bitcoin market, respectively. r is the threshold parameter which partitions the state space of the past return R_{t-1} with a view to classifying to two regimes in the Bitcoin market. Suppose $\{R_{t-1} \geq r\}$ and $\{R_{t-1} < r\}$ represent "Good" and "Bad" regimes, respectively, in the Bitcoin market in the period from time $t-1$ to time t; α_0, α_1, β and γ are the parameters in the GARCH component of the model such that $\alpha_0 > 0$, $\alpha_1 \geq 0$ and $\beta \geq 0$. The parameter γ determines the correlation between the conditional variance and the return. When $\gamma > 0$, the correlation between the conditional variance and the return is negative.

The parameters to be estimated in the SETAR-GARCH model in Eq. (2.1)-Eq. (2.2) are λ_1, λ_2, α_0, α_1, β and γ. One may consider some generalizations of the SETAR-GARCH model in Eq. (2.1)-Eq. (2.2). For example, more than two regimes in the SETAR component may be considered; time delay in the SETAR component may be incorporated using R_{t-d} rather than R_{t-1} in classifying the regimes, where d is a time delay parameter; conditional non-normality for the innovations may be studied. Time-varying interest rates may be used. However, to focus on the impacts of conditional heteroscedasticity and regime switches on Bitcoin option prices, a simple SETAR-GARCH model described in Eq. (2.1)-Eq. (2.2) is adopted.

Remark 1. There are two components in the SETAR-GARCH model described in Eq. (2.1)-Eq. (2.2). The first component is the conditional-mean component, which is given by Eq. (2.1). This component is modelled by a two-regime, first-order, SETAR model and describes the regime-switching behavior of the conditional mean of the logarithmic return given the information $\mathcal{F}_{t-1}$ up to and including time $t-1$, say $\mathrm{E}[R_t|\mathcal{F}_{t-1}]$. Specifically, when $R_{t-1} \geq r$ (i.e., the Bitcoin market is in a "Good" regime in the period from time $t-1$ to time t), the conditional mean $\mathrm{E}[R_t|\mathcal{F}_{t-1}]$ is given by $r_d - r_s + (\lambda_1 - \frac{1}{2})h_t$. Whereas, when $R_{t-1} < r$ (i.e., the Bitcoin market is in a "Bad" regime in the period from time $t-1$ to time t), the conditional mean $\mathrm{E}[R_t|\mathcal{F}_{t-1}]$ is given by $r_d - r_s + (\lambda_2 - \frac{1}{2})h_t$. See Tong (1990) for more detailed discussions on the SETAR model. The second component is the conditional variance component, which is given by Eq. (2.2). This component is modeled by the Heston-Nandi GARCH(1,1) model and describes time-varying conditional volatility (i.e., conditional heteroscedasticity). From Eq. (2.2), the conditional variance h_t at time t depends on the conditional variance h_{t-1} at time $t-1$ and the square of the random error term ϵ_{t-1}^2 at time $t-1$. See Taylor (2005) for more detailed discussions on the GARCH model.

The conditional Esscher transform was used to select a pricing kernel in Siu et al. (2006) under the SETAR-ARCH model. It was used in Siu et al. (2004) to select a pricing kernel under the Heston-Nandi GARCH model in Heston and Nandi (2000). It was also adopted by Elliott et al. (2006) to select a pricing kernel for a Markov-switching Heston-Nandi GARCH model. Here we adapt the pricing methodology based on the conditional Esscher transform in Siu et al. (2004, 2006) and Elliott et al. (2006) to select a pricing kernel for the SETAR-GARCH model with the Heston-Nandi GARCH component. Alternatively, one may explore the possibility of using a locally risk-neutral valuation principle for GARCH option pricing proposed by Duan (1995) in the SETAR-GARCH set up. The uses of the Esscher transform and conditional Esscher transform for option pricing were proposed in Gerber and Shiu (1994) and Bühlmann et al. (1996), respectively. See also Gerber and Shiu (2019) for a recent account on the use of the Esscher transform, which is an important tool in actuarial science, for valuation. The pricing kernel selected by the conditional Esscher transform was justified by the expected utility maximization under GARCH and SETAR models in, for example, Siu et al. (2004, 2006), respectively, which followed the arguments in Gerber and Shiu (1994). Specifically, in Gerber and Shiu (1994), the justification for a pricing kernel selected by the Esscher transform was based on the no-trade condition for an option position which may be related to a variational argument. See, for example, Gerber and Shiu (2000) and Siu et al. (2004) for some related discussions. Bühlmann (1980, 1984) provided a justification for the use of the Esscher transform for insurance premium principles using an economic equilibrium approach. Only key steps for specifying a pricing kernel are provided here. For details, one may refer to

Siu et al. (2004, 2006) and Elliott et al. (2006).

Consider an $(\mathbb{F}, \mathbb{P})$-martingale $\{\Lambda_t\}_{t \in \mathcal{T}}$ defined by the conditional Esscher transform as follows:

$$\Lambda_t = \prod_{k=1}^{t} \frac{e^{\theta_k R_k}}{M_R(\theta_k)}, \quad \Lambda_0 = 1, \quad \mathbb{P}\text{-a.s.}, \tag{2.3}$$

where the process $\{\theta_t\}_{t \in \mathcal{T}_1}$ is $\mathbb{F}$-predictable; $M_R(\theta_t) := \mathrm{E}[e^{\theta_t R_t} | \mathcal{F}_{t-1}]$, which is the conditional moment generating function of R_t given $\mathcal{F}_{t-1}$ under $\mathbb{P}$ evaluated at θ_t; $\mathrm{E}[\cdot]$ is the expectation under $\mathbb{P}$; it is supposed that $M_R(\theta_t) < \infty$. For convenience, we take $\mathbb{F}$ to be the $\mathbb{P}$-completed natural filtration generated by the return process $\{R_t\}_{t \in \mathcal{T}_1}$, where $\mathcal{F}_0 = \sigma\{\emptyset, \Omega\}$.

A new probability measure $\mathbb{P}^\theta$ equivalent to $\mathbb{P}$ on $\mathcal{F}_t$, for each $t \in \mathcal{T}$, is defined by putting:

$$\left. \frac{d\mathbb{P}^\theta}{d\mathbb{P}} \right|_{\mathcal{F}_t} := \Lambda_t. \tag{2.4}$$

To preclude arbitrage opportunities, using the fundamental theorem of asset pricing in Harrison and Kreps (1979) and Harrison and Pliska (1981, 1983), the discounted price process of the Bitcoin index, say $\{e^{-(r_d - r_s)t} S_t\}_{t \in \mathcal{T}}$, should be an $(\mathbb{F}, \mathbb{P}^\theta)$-martingale. This is a martingale condition. Here we shall adopt a risk-neutral valuation methodology based on this martingale condition and the conditional Esscher transform to select a risk-neutral, or martingale, probability measure for valuing Bitcoin options.

Remark 2. Λ_t is related to the pricing kernel or stochastic discount factor in asset pricing. $\left. \frac{d\mathbb{P}^\theta}{d\mathbb{P}} \right|_{\mathcal{F}_t}$ in Eq. (2.4) may be thought of as the likelihood ratio given $\mathcal{F}_t$ between the two probability measures $\mathbb{P}^\theta$ and $\mathbb{P}$. In other words, the likelihood ratio $\left. \frac{d\mathbb{P}^\theta}{d\mathbb{P}} \right|_{\mathcal{F}_t}$ is specified by Λ_t. The probability measure $\mathbb{P}^\theta$ is the one to be used for valuing Bitcoin options. It is also called the pricing probability measure. The price of a Bitcoin option is evaluated by taking the expectation on the payoff function of the option with respect to the pricing probability measure $\mathbb{P}^\theta$, where $\theta = \theta^*$ satisfying the martingale condition to preclude arbitrage opportunities. The martingale condition will be presented in the sequel. To evaluate the expectation on the payoff function of the option under the pricing probability measure, the dynamics for the logarithmic returns and conditional variances under the pricing probability measure are used. These dynamics will be presented in the sequel.

Jarrow (2012) established the equivalence between the existence of an equivalent martingale measure and the informational efficiency of a market. This is called the third fundamental theorem of asset pricing which implies that in a complete market, the validity of risk-neutral derivative valuation and the information efficiency of a market are equivalent. Here we do not consider a complete market, but an equivalent martingale measure selected by the conditional Esscher transform will be used for valuing Bitcoin options. The third fundamental theorem of asset pricing in Jarrow (2012) may provide some insights into understanding the relationship between risk-neutral valuation of Bitcoin options based on the equivalent martingale measure and the informational efficiency of Bitcoin markets. There have been some empirical works on testing the market efficiency of Bitcoins. The empirical results seem to be mixed. For example, Urquhart (2016) presented empirical evidence for the informational inefficiency of Bitcoin markets, but also found that the efficiency of Bitcoin markets is improving as time goes by. Bariviera (2017) found that daily Bitcoin returns have become more informationally efficient since 2014. Nadarajah and Chu (2017) found the weak-form efficiency of a power transformation of Bitcoin returns. Tiwari et al. (2018) consolidated some findings of Urquhart (2016), Bariviera (2017) and Nadarajah and Chu (2017) by reporting the informational efficiency of Bitcoin markets, except during the periods from April 2013 to August 2013 and from August 2016 to November 2016. Bariviera et al. (2017) found long memory in Bitcoin returns, and hence, provided empirical evidence against informational efficiency of Bitcoin returns. See also Silva de Souza et al. (2019) for some discussions for the informational efficiency of Bitcoin markets. In a recent study by Wei (2018) based on 456 cryptocurrencies, it was found that, though Bitcoin returns exhibit signs of efficiency, returns from many cryptocurrencies show signs of autocorrelation. Wei (2018) seemed to conclude that the informational efficiency of Bitcoin returns is stronger when the liquidity of Bitcoin markets is higher. Based on these empirical results and the third fundamental theorem of asset pricing, it may appear that the validity of risk-neutral valuation of Bitcoin options may be related to the liquidity of Bitcoin markets, where as noted in Wei (2018), active traders in liquid Bitcoin markets may arbitrage away profits from predictability of Bitcoin returns. This may then eliminate arbitrage opportunities. Nevertheless, there seems to be mixed results about this. For example, Trimborn et al. (2019) noted that cryptocurrencies have a lower level of liquidity than traditional assets. Makarov and Schoar (2019a) found that arbitrage opportunities are substantial across exchanges though price deviations are much more significant across than within countries. Their findings appear to question the validity of the law of one price in Bitcoin markets. Makarov and Schoar (2019a) also pointed out that arbitrage opportunities and spreads may be significantly constrained by cross-border capital controls and other regulations on trading constraints. These are important practical considerations when applying the risk-neutral valuation methodology for valuing Bitcoin options. There is a need to bear in mind the limitations of the use of the risk-neutral valuation methodology for valuing Bitcoin options. Specifically, the no-arbitrage assumption underlying the risk-neutral valuation methodology for evaluating theoretical prices of Bitcoin options may not hold in real Bitcoin markets. In a paper by Delbaen and Hazendonck (1989), an approach based on an equivalent martingale probability measure in, for example, Harrison and Kreps (1979) and Harrison and Pliska (1981, 1983), was used for premium calculations in an arbitrage-free market. They noted that the martingale approach based on no-arbitrage may be applied for a liquid insurance market. The relationship between a liquid insurance market and the correctness of a "fair" price of an insurance product was also noted in Bühlmann et al. (1996), where the use of the conditional Esscher transform for no-arbitrage pricing was discussed. The martingale approach, which is based on the absence of arbitrage and a risk-neutral probability measure, is used here to value Bitcoin options. This approach is based on the premise of the law of one price and was adopted in Hou et al. (2019) for valuing cryptocurrency options under stochastic volatility correlated jumps models. However, the conditional Esscher transform is applied here to select a martingale probability measure for valuing Bitcoin options.

Note that the martingale condition implies:

$$\theta_t^* = -\lambda_1 I_{\{R_{t-1} \geq r\}} - \lambda_2 I_{\{R_{t-1} < r\}}. \tag{2.5}$$

where I_A is the indicator function of an event A.

Eq. (2.5) gives the risk-neutral conditional Esscher parameter at time t for the SETAR-GARCH model. Consequently, the pricing kernel is specified by a risk-neutral probability measure $\mathbb{P}^{\theta^*}$. Intuitively, it may appear that the liquidity in a "Good" market regime may be higher than that in a "Bad" market regime. Consequently, the difference in the unit risk premiums in the two market regimes, say $\lambda_1 - \lambda_2$, may be related to the liquidity premium due to changes in regimes in Bitcoin markets. In this vein, it may be seen that the risk-neutral conditional Esscher parameter in Eq. (2.5) may take into account these changes in regimes in the valuation of Bitcoin options. It can be shown that under $\mathbb{P}^{\theta^*}$, the conditional distribution of the logarithmic return R_t given $\mathcal{F}_{t-1}$ follows a normal distribution with mean $r_d - r_s - \frac{1}{2}h_t$ and variance h_t. Then,

$$R_t = r_d - r_s - \frac{1}{2}h_t + \sqrt{h_t}\epsilon_t^*, \tag{2.6}$$

for some process $\{\epsilon_t^*\}_{t \in \mathcal{T}_1}$ such that $\epsilon_t^* | \mathcal{F}_{t-1} \sim N(0,1)$ under $\mathbb{P}^{\theta^*}$.

For each $t \in \mathcal{T}_1$, let $\lambda(t) := \lambda_1 I_{\{R_{t-1} \geq r\}} + \lambda_2 I_{\{R_{t-1} < r\}}$. Then the return process in Eq. (2.1) becomes:

$$R_t = r_d - r_s + \left(\lambda(t) - \frac{1}{2}\right)h_t + \sqrt{h_t}\epsilon_t. \tag{2.7}$$

As in Duan (1995), Proof of Theorem 2.2 on Page 27, by comparing the return processes in Eq. (2.6) and Eq. (2.7), it can be seen that

$$\epsilon_t = \epsilon_t^* - \lambda(t)\sqrt{h_t}. \tag{2.8}$$

Substituting ϵ_t in Eq. (2.8) into the conditional variance process in Eq. (2.2), the conditional variance process under $\mathbb{P}^{\theta^*}$ becomes:

$$h_t = \alpha_0 + \alpha_1(\epsilon_{t-1}^* - (\gamma + \lambda(t-1))\sqrt{h_{t-1}})^2 + \beta h_{t-1}, \tag{2.9}$$

where $\lambda(t-1) = \lambda_1 I_{\{R_{t-2} \geq r\}} + \lambda_2 I_{\{R_{t-2} < r\}}$.

Then the conditional price V_t of a European-style Bitcoin option with maturity T and payoff V_T at time T given $\mathcal{F}_t$ is:

$$V_t = \mathrm{E}^{\theta^*}[e^{-(r_d - r_s)(T-t)} V_T | \mathcal{F}_t], \tag{2.10}$$

where $\mathrm{E}^{\theta^*}[\cdot]$ is the expectation under $\mathbb{P}^{\theta^*}$.

The two other models used for comparison are the version of the Heston-Nandi GARCH(1, 1) model in Christoffersen et al. (2013) and the Black-Scholes-Merton model. Under the version of the Heston-Nandi GARCH(1,1) model and the real-world probability measure $\mathbb{P}$, the return process $\{R_t\}_{t \in \mathcal{T}_1}$ and the conditional variance process

$\{h_t\}_{t \in \mathcal{T}_1}$ follow:

$$\begin{aligned} R_t &= r_d - r_s + \left(\lambda - \frac{1}{2}\right) h_t + \sqrt{h_t}\epsilon_t, \\ h_t &= \alpha_0 + \alpha_1(\epsilon_{t-1} - \gamma\sqrt{h_{t-1}})^2 + \beta h_{t-1}. \end{aligned} \quad (2.11)$$

Note that the risk premium λ in the version of the Heston-Nandi GARCH(1,1) model in Christoffersen et al. (2013) is related to the risk premium λ_{HN} in the version of the Heston-Nandi GARCH(1,1) model in Heston and Nandi (2000) as follows:

$$\lambda = \lambda_{HN} + \frac{1}{2}. \quad (2.12)$$

As in Siu et al. (2004), under a risk-neutral probability measure $\mathbb{P}^{\theta^*}$ selected by the conditional Esscher transform, the return process and the conditional variance process under the Heston-Nandi GARCH(1,1) model become:

$$\begin{aligned} R_t &= r_d - r_s - \frac{1}{2} h_t + \sqrt{h_t}\epsilon_t^*, \\ h_t &= \alpha_0 + \alpha_1(\epsilon_{t-1}^* - (\gamma + \lambda)\sqrt{h_{t-1}})^2 + \beta h_{t-1}, \end{aligned} \quad (2.13)$$

where $\epsilon_t^* | \mathcal{F}_{t-1} \sim N(0, 1)$ under $\mathbb{P}^{\theta^*}$.

Remark 3. The risk-neutral dynamics for the logarithmic returns and the conditional variances under the pricing probability measure $\mathbb{P}^{\theta^*}$ will be used to compute the "risk-neutral" expectation in Eq. (2.10) under $\mathbb{P}^{\theta^*}$ for computing the price of the European-style Bitcoin option. This will be done by Monte-Carlo simulation in the next section.

In the sequel, we shall consider a variance-dependent pricing kernel. The key idea of the variance-dependent pricing kernel is to incorporate a variance component in the specification of a pricing kernel. The variance risk in the variance-dependent pricing kernel is priced explicitly by introducing the market price of variance risk or the variance risk premium. The significance of using the variance-dependent pricing kernel to account for some empirical anomalies of option prices data was emphasized in an important paper by Christoffersen et al. (2013). Some other works on the variance-dependent pricing kernel include, for example, Corsi et al. (2013), Christoffersen et al. (2015), Majewski et al. (2015), Bormetti et al. (2016), Badescu et al. (2017, 2019), Bormetti et al. (2019), Alitab et al. (2020). As noted in Badescu et al. (2017), the variance-pricing pricing kernel can be thought of as an extension to a pricing kernel specified by the conditional Esscher transform, which is also in an exponential affine form. To simplify our discussion, we shall consider the use of the variance-dependent pricing kernel for valuation European-style Bitcoin options under the the Heston-Nandi GARCH(1,1) model with normal innovations in Eq. (2.11). We shall examine, in the next section, the impacts of introducing the variance component in the specification of a pricing kernel under the Heston-Nandi GARCH(1,1) model with normal innovations on the European call and put Bitcoin option prices and their implied volatilities. To discuss the variance-dependent pricing kernel and the risk-neutral price dynamics under the Heston-Nandi GARCH(1,1) model with normal innovations in Eq. (2.11), we follow Christoffersen et al. (2013).

Firstly, following Christoffersen et al. (2013), Page 1999, the variance-dependent pricing kernel is an $\mathbb{F}$-adapted exponential process $\{M_t | t \in \mathcal{T}_1\}$, which is defined as follows:

$$M_t = M_{t-1} \exp(\delta + \phi R_t + \eta h_t + \xi(h_{t+1} - h_t)), \tag{2.14}$$

where

$$\begin{aligned}
\delta &:= -(\phi+1)r - \xi\alpha_0 + \frac{1}{2}\ln(1-2\xi\alpha_1), \\
\eta &:= -(\lambda - \frac{1}{2})\phi - \xi\alpha_1\gamma + (1-\beta)\xi - \frac{(\phi - 2\xi\alpha_1\gamma)^2}{2(1-2\xi\alpha_1)}, \\
\phi &:= -(\lambda - \frac{1}{2} + \gamma)(1 - 2\alpha_1\xi) + \gamma - \frac{1}{2}.
\end{aligned} \tag{2.15}$$

From Proposition 1 of Christoffersen et al. (2013), under the transformation by the variance-dependent pricing kernel in Eq. (2.14), the risk-neutral return process and conditional variance process corresponding to the version of the Heston-Nandi GARCH(1,1) model in Eq. (2.11) are given by:

$$\begin{aligned}
R_t &= r_d - r_s - \frac{1}{2}h_t^* + \sqrt{h_t^*}z_t^*, \\
h_t^* &= \alpha_0^* + \alpha_1^*(z_{t-1}^* - \gamma^*\sqrt{h_{t-1}^*})^2 + \beta h_{t-1}^*,
\end{aligned} \tag{2.16}$$

where $\{z_t^*\}_{t\in\mathcal{T}_1}$ is a sequence of independent and identically distributed standard normal variables. Furthermore, the risk-neutral conditional variance process $\{h_t^*\}_{t\in\mathcal{T}_1}$ and the risk-neutral parameters α_0^*, α_1^* and γ^* are related to their real-world counterparts as follows:

$$\begin{aligned}
h_t^* &= \frac{h_t}{1-2\alpha_1\xi}, \\
\alpha_0^* &= \frac{\alpha_0}{1-2\alpha_1\xi}, \\
\alpha_1^* &= \frac{\alpha_1}{(1-2\alpha_1\xi)^2}, \\
\gamma^* &= \gamma - \phi.
\end{aligned} \tag{2.17}$$

When $\xi = 0$, $\phi = -\lambda$ by Eq. (2.15). Also, from Eq. (2.17), $h_t^* = h_t$, $\alpha_0^* = \alpha_0$, $\alpha_1^* = \alpha_1$ and $\gamma^* = \gamma + \lambda$. Consequently, the risk-neutral return process and conditional variance process in Eq. (2.16) corresponding to the variance-dependent pricing kernel in Eq. (2.14) coincide with those selected by the conditional Esscher transform in Eq. (2.13). It was noted in Christoffersen et al. (2013), Corollary 3 on Page 1970, that the variance risk premium is negative when the variance preference parameter ξ is positive.

From Christoffersen et al. (2013), Page 1973, the unconditional risk-neutral variance for the Heston-Nandi GARCH(1,1) model in Eq. (2.16) under the variance-dependent pricing kernel is given by:

$$\mathrm{E}^{\mathbb{Q}}[h_t^*] = \frac{\alpha_0^* + \alpha_1^*}{1 - \beta - \alpha_1^*\gamma^{*2}}, \tag{2.18}$$

where $\mathrm{E}^{\mathbb{Q}}$ is the expectation under the risk-neutral probability measure corresponding to the variance-dependent pricing kernel in Eq. (2.14); h_t^* is the conditional variance process in Eq. (2.16); α_0^*, α_1^* and γ^* are the risk-neutral parameters in Eq. (2.17). When $\xi = 0$, Eq. (2.18) becomes the unconditional risk-neutral variance for the Heston-Nandi GARCH(1, 1) model in Eq. (2.13), which is given by:

$$\mathrm{E}^{\theta}[h_t] = \frac{\alpha_0 + \alpha_1}{1 - \alpha_1(\gamma + \lambda)^2 - \beta}. \tag{2.19}$$

The unconditional risk-neutral variance for the SETAR-GARCH model in Eq. (2.9) is given by:

$$\mathrm{E}^{\theta^*}[h_t] = \frac{\alpha_0 + \alpha_1}{1 - \beta - \alpha_1((\gamma + \lambda_1)^2 \pi + (\gamma + \lambda_2)^2 (1 - \pi))}, \tag{2.20}$$

where $\pi := \mathbb{P}^{\theta^*}(R_t \geq r)$. See, for example, Proposition 2 of Wu (2011) for a related result for a threshold GARCH model. Heuristic derivations for Eq. (2.20) under the assumption for covariance stationarity are presented in the Appendix. When there is a single regime (i.e., $\pi = 1$ and $\lambda_1 = \lambda_2 = \lambda$), the unconditional risk-neutral variance in Eq. (2.20) becomes the unconditional risk-neutral variance in Eq. (2.19) for the Heston-Nandi GARCH(1, 1) model in Eq. (2.13).

An option pricing formula for a European call option based on the conditional moment generating function or the conditional characteristic function of the terminal (logarithmic) price of the underlying asset under the Heston-Nandi GARCH model was provided in Heston and Nandi (2000). See Christoffersen et al. (2013) for the use of the Heston-Nandi option pricing formula for the case of a variance-dependent pricing kernel. Here we adopt the version of the Heston-Nandi option pricing formula in Christoffersen et al. (2013) corresponding to the variance-dependent pricing kernel, which is presented in the sequel. Let $\{S_t\}_{t \in \mathcal{T}}$ denote the Bitcoin price process and $g_{t,T}^*(\kappa)$ be the conditional moment generating function of terminal (logarithmic) price of the Bitcoin at time T under the risk-neutral probability measure corresponding to the variance-dependent pricing kernel given $\mathcal{T}$ evaluated at κ. Then

$$\begin{aligned} g_{t,T}^*(\kappa) &:= \mathrm{E}^{\mathbb{Q}}[\exp(\kappa \ln(S_T)) | \mathcal{F}_t] \\ &= \exp(\kappa \ln(S_t) + A_{t,T}(\kappa) + B_{t,T}(\kappa) h_{t+1}^*), \end{aligned} \tag{2.21}$$

where the coefficients $A_{t,T}(\kappa)$ and $B_{t,T}(\kappa)$ satisfy the following system of backward difference equations:

$$\begin{aligned} A_{t,T}(\kappa) &= A_{t+1,T}(\kappa) + \kappa r + B_{t+1,T}(\kappa)\alpha_0^* - \frac{1}{2}\ln(1 - 2B_{t+1,T}(\kappa)\alpha_1^*), \\ B_{t,T}(\kappa) &= -\frac{1}{2}\kappa + B_{t+1,T}(\kappa)\beta + B_{t+1,T}(\kappa)\alpha_1^* \gamma^{*2} \\ &\quad + \frac{\frac{1}{2}\kappa^2 + 2B_{t+1,T}(\kappa)\alpha_1^*\gamma^*(B_{t+1,T}(\kappa)\alpha_1^*\gamma^* - \kappa)}{1 - 2B_{t+1,T}(\kappa)\alpha_1^*}, \end{aligned} \tag{2.22}$$

with the terminal condtion $A_{T,T}(\kappa) = B_{T,T}(\kappa) = 0$.

The time-t price of a European call option with strike price K and maturity T is given by:

$$C(t, S_t, h_{t+1}) = S_t \Pi_{1t} - K e^{-r(T-t)} \Pi_{2t}, \tag{2.23}$$

where

$$\Pi_{1t} = \frac{1}{2} + \frac{e^{-r(T-t)}}{\pi} \int_0^{\infty} \mathrm{Re}\left[\frac{K^{-i\kappa} g^*_{t,T}(i\kappa + 1)}{i\kappa S_t}\right] d\kappa,$$
$$\Pi_{2t} = \frac{1}{2} + \frac{1}{\pi} \int_0^{\infty} \mathrm{Re}\left[\frac{K^{-i\kappa} g^*_{t,T}(i\kappa)}{i\kappa}\right] d\kappa, \quad (2.24)$$

where $\mathrm{Re}(z)$ is the real part of a complex number z and $i = \sqrt{-1}$. When $\xi = 0$, the pricing formula in Eq. (2.23)-Eq. (2.24) becomes the one for the risk-neutral Heston-Nandi GARCH(1,1) model under the conditional Esscher transform in Eq. (2.13). In the next section, the pricing formula in Eq. (2.23)-Eq. (2.24) will be used to compute the European call Bitcoin prices from the risk-neutral Heston-Nandi GARCH(1,1) models under the variance-dependent pricing kernel and the conditional Esscher transform. The Monte Carlo simulation will be used to compute the European call Bitcoin prices from the SETAR-GARCH model with the Heston-Nandi GARCH(1,1) component.

3. Numerical Studies and Comparison

Numerical studies for Bitcoin option prices from the SETAR-Heston-Nandi-GARCH, Heston-Nandi GARCH and Black-Scholes-Merton models estimated using real price data on Bitcoin price index, say the CoinDesk index, are provided. Specifically, the daily close prices of the CoinDesk index in the U.S dollar from 18 July 2010 to 31 May 2018 (with a total of 2875 observations) were used. We shall also provide numerical studies for the impacts of introducing the variance component in the specification for a pricing kernel under the Heston-Nandi GARCH model with normal innovations in Eq. (2.11) on the European call and put Bitcoin option prices and their implied volatilities. Specifically, these impacts are studied through varying the parameter ξ relating to the variance premium, where a positive ξ corresponds to a negative variance premium. The data were downloaded from the CoinDesk index page `https://www.coindesk.com/price/`. The (conditional) maximum likelihood estimation was used to estimate the SETAR-GARCH and GARCH models. For the estimation of the threshold parameter in the SETAR-Heston-Nandi-GARCH model, 11 percentile points were used as the candidate values of the threshold parameter. This method was applied in Siu (2019a) to estimate the threshold parameter in a SETAR model in a Bayesian nonlinear expectation framework. Monte Carlo simulations will be adopted to compute the price matrices for European call and put Bitcoin options with different strikes and maturities from the estimated SETAR-Heston-Nandi-GARCH model. Each option price was computed using 10,000 simulation runs. For the computations of the price matrices for European call and put Bitcoin options from the estimated Heston-Nandi GARCH model under both the conditional Esscher transform, (i.e., the variance-dependent pricing kernel with $\xi = 0$), and the variance-dependent pricing kernel, the pricing formula in Eq. (2.23)-Eq. (2.24) is used. The initial values, say the current Bitcoin price, the current conditional variance, the current Bitcoin logarithmic return and the Bitcoin logarithmic return on the previous day, used in the simulations were \$7488.79, 0.00342206, 0.01522049 and -0.01144641, respectively [2]. The domestic interest rate r_d was calculated using the interest rate from Treasury

[2]The current Bitcoin price was taken as the close price of the CoinDesk index on 31 May 2018; the current conditional variance was estimated by the sample variance of the daily logarithmic returns from the Coindesk

bills. Specifically, the Treasury Bill Rate for 13 weeks bank discount on 31 May 2018, which was 1.89% (a 360-day year), was used. Consequently, $r_d = 5.25 \times 10^{-5}$. It was noted in Bariviera et al. (2017), page 84, that there are no interest rates on Bitcoins. Consequently, the Bitcoin interest rate is supposed to be zero here. That is, $r_s = 0$. All the computations were done by R codes. In particular, the R codes for estimating the SETAR-Heston-Nandi-GARCH and Heston-Nandi GARCH models were developed by modifying the R codes in Chalabi and Würtz (2008), where the R function "nlminb" was used in the optimization. The computations of the European call and put Bitcoin option prices using the pricing formula in Eq. (2.23)-Eq. (2.24) were done by using the function "HNGOption" in the R package "fOptions" (Wuertz et al. (2017)). Most of the tables to be presented below were generated from stargazer by Hlavac (2018).

Table A.1 below gives the estimation results for the SETAR-Heston-Nandi-GARCH and Heston-Nandi GARCH models. The standard errors of the parameters estimates, except the threshold parameter, are presented in curly brackets below the respective parameters estimates. Some initial estimates and tolerance levels for the GARCH parameters in the SETAR-Heston-Nandi-GARCH and Heston-Nandi GARCH models followed those used in Chalabi and Würtz (2008). Say the initial estimates for the GARCH parameters are: $\alpha_0 = 0.1 \times s_R^2$, $\alpha_1 = 0.1$ and $\beta = 0.8$, where s_R^2 is the sample variance of the daily logarithmic returns from the CoinDesk index data. The initial estimate for γ is 1.0 for both the SETAR-Heston-Nandi-GARCH and Heston-Nandi GARCH models. The initial estimates for the unit risk premiums in the two regimes of the SETAR-Heston-Nandi-GARCH model are: $\lambda_1 = 0.005$ and $\lambda_2 = 0.001$. The initial estimate for the unit risk premium in the Heston-Nandi GARCH model is: $\lambda = 0.005$. For the initial values of the conditional variances, the sample variance of the daily logarithmic returns from the CoinDesk index data was used.

$\sim$ Table A.1 about here $\sim$

From Table A.1, it can be seen that the estimates for the parameters in the GARCH equations, say α_0, α_1 and β, in both the SETAR-Heston-Nandi-GARCH and Heston-Nandi GARCH models appear to be precise, (i.e., the standard errors are small relative to the parameters estimates). However, the estimate for γ is much less precise. The estimates for the unit risk premiums in the two models, say λ_1, λ_2 and λ, seem to be less precise compared with the estimates for α_0, α_1 and β. This seems to be in line with Merton (1980), where it was noted that the estimation for expected rates of return for financial assets was imprecise. Furthermore, compared with the estimates for λ_2 and λ, the estimate for λ_1 is much less precise. Comparing the negative log-likelihood values of the two models, it looks that the SETAR-GARCH model may provide a slightly better fit to the CoinDesk returns data than the GARCH model, (i.e., the former has a slightly smaller value for the negative log-likelihood value than the latter). To test for the presence of the SETAR regime switching effect in the daily logarithmic returns in the CoinDesk index data, the statistical test of linearity against threshold with bootstrap distribution in Hansen (1999) is conducted using the function "setarTest" in the R package "tsDyn" (Di Narzo et al. (2019)). The test statistics for the test of linearity against a two-regime SETAR model and a three-regime SETAR model are, respectively, 37.28176 and 56.68053. The p-values of the two test statistics are zero, (i.e., very close to zero). This may provide evidence for the presence of the SETAR

index data; the daily logarithmic returns on the current day and the previous day were computed as the respective returns on 31 May 2018 and 30 May 2018 from the CoinDesk index data.

regime switching effect in the returns data of the CoinDesk index [3].

Table A.2 below provides the estimated unconditional risk-neutral variances under the SETAR-Heston-Nandi-GARCH model, the Heston-Nandi GARCH model using the conditional Esscher transform, (i.e., the variance-dependent pricing kernel with $\xi = 0$), the Heston-Nandi GARCH model using the variance-dependent pricing kernel with $\xi = 100, 200, 300$. Note that the unconditional risk-neutral variance under the SETAR-Heston-Nandi-GARCH model was computed using Eq. (2.20), where the regime probability $\pi := \mathbb{P}^{\theta^*}(R_t \geq r)$ was approximated by counting the empirical relative frequency of R_t exceeding the threshold parameter r. The unconditional risk-neutral variance under the Heston-Nandi GARCH model using the conditional Esscher transform was computed using Eq. (2.19), while the unconditional risk-neutral variances under the Heston-Nandi GARCH model using the variance-dependent pricing kernel were computed using Eq. (2.18).

~ Table A.2 about here ~

From Table A.2, it can be seen that all the unconditional risk-neutral variances are positive. This indicates the covariance stationarity of the estimated risk-neutral SETAR-Heston-Nandi-GARCH model, the estimated risk-neutral Heston-Nandi GARCH model using the conditional Esscher transform and the estimated risk-neutral Heston-Nandi GARCH models using the variance-dependent pricing kernel.

Tables A.3-A.4 below present the Monte Carlo estimates of the prices for the European call and put Bitcoin options, respectively, as well as their standard errors (in the curly brackets below the estimates) with different strikes and maturities from the estimated SETAR-Heston-Nandi-GARCH model.

~ Tables A.3-A.4 about here ~

From Table A.3, it can be seen that the Monte Carlo estimates of the prices of the European call Bitcoin options are consistent with financial intuition in most of the cases. Specifically, when the strike price increases, (i.e., the moneyness changes from in-the-money (ITM) to out-of-the-money (OTM) through at-the-money (ATM)), the European call Bitcoin option price mostly decreases, except for a few cases. As the time to maturity increases from 30 days to 360 days, the European call Bitcoin option price mostly increases, except for a few cases. This reflects that the time value of the call option mostly increases as the time to maturity does.

From Table A.4, the Monte Carlo estimates for the prices of the European put Bitcoin options appear to make financial sense. Say when the strike price increases, (i.e., the moneyness changes from OTM to ITM through ATM), the European put Bitcoin option price increases. The standard errors for the Monte Carlo estimates for the prices of the European put Bitcoin options are relatively small compared with the respective price estimates. This may indicate that the Monte Carlo estimates for the prices of the European put Bitcoin options appear to be reasonably precise.

Tables A.5-A.6 below present the prices for the European call and put Bitcoin options, respectively, with different strikes and maturities from the estimated Heston-Nandi GARCH model, where the pricing kernel is specified by the conditional Esscher transform, or equivalently, the variance-dependent pricing kernel with $\xi = 0$.

[3] Siu (2019a) conducted the statistical test of Hansen (1999) to the daily percentage logarithmic returns from a Bitcoin exchange rate series in the U.S. dollar, namely "BitStamp", and found evidence for the use of a two-regime SETAR model.

∼ Tables A.5-A.6 about here ∼

From Tables A.5-A.6, it can be observed that the prices of the European call and put Bitcoin options under the estimated Heston-Nandi GARCH model have similar patterns with the estimates for the respective prices under the estimated SETAR-Heston-Nandi-GARCH model presented in Tables A.3-A.4, as the strike price or the time to maturity increases.

Tables A.7-A.8 below give the prices for the European call and put Bitcoin options, respectively, with different strikes and maturities from the Black-Scholes-Merton model. Note that the annualized interest rates used are $r_d \times 360 = 0.0189$ and $r_s \times 360 = 0$, while the annualized volatility used is $s_R \times \sqrt{360} = 1.109929$, where s_R is the sample standard deviation of the daily logarithmic returns from the CoinDesk index data. The current Bitcoin price used is \$7488.79. Comparing the results in Tables A.7-A.8 with those in Tables A.3-A.6, it appears that the impact of conditional heteroscedasticity on the Bitcoin option prices is quite significant.

∼ Tables A.7-A.8 about here ∼

The prices for the European call and put Bitcoin options from the Black-Scholes-Merton model in Tables A.7-A.8 are used as a benchmark for comparison. The overall patterns of the call and put prices from the Black-Scholes-Merton model, as the strike price or the time to maturity varies, appear to be similar to those of the respective prices from the estimated SETAR-Heston-Nandi-GARCH and Heston-Nandi GARCH models presented in Tables A.3-A.4 and Tables A.5-A.6, respectively.

Figures B.1-B.2 below plot the prices of the European call and put Bitcoin options, respectively, under the SETAR-Heston-Nandi-GARCH model and the Heston-Nandi GARCH model against maturity for the in-the-money (ITM), at-the-money (ATM) and out-of-the-money (OTM) options.

∼ Figures B.1-B.2 about here ∼

From Figures B.1-B.2, it may be seen that the prices of the European call and put Bitcoin options under the SETAR-Heston-Nandi-GARCH model and the Heston-Nandi GARCH model are close to each other. This seems to indicate that the impact of regime switches described by the SETAR component on the European call and put Bitcoins is not significant. This observation may apparently look counter-intuitive given that the results of the statistical test may provide empirical evidence for the presence of the SETAR regime switching effect in the CoinDesk index returns data. However, through the risk neutralization via the conditional Esscher transform, the regime switching effect described by the SETAR model in the conditional mean rate of return is mitigated to the conditional variance equation, see Eq. (2.6) and Eq. (2.9). Furthermore, it may be possible that the impact of conditional heteroscedasticity attributed to the GARCH effect on the option prices may dominate that of the regime switching effect described by the SETAR component on the option prices.

Figure B.3 below depicts the plots of the Black-Scholes implied volatilities for European Bitcoin call options against moneyness from the SETAR-Heston-Nandi-GARCH model and the Heston-Nandi GARCH model for three different maturities. The Black-Scholes implied volatilities were computed using the R function "compute.implied.volatility" in the R package "RND" (Hamidieh (2017)).

∼ Figure B.3 about here ∼

From Figure B.3, it appears that the the Black-Scholes implied volatilities for the European Bitcoin call option prices simulated from the SETAR-Heston-Nandi-GARCH model slightly exhibit implied volatility skew for short-maturity (30 days) options though there are some fluctuations. It also seems that the Black-Scholes implied volatilities for the European Bitcoin call option prices evaluated from the Heston-Nandi GARCH model slightly exhibit implied volatility skew for short-maturity (30 days) options. This appears to be in line with the results obtained in Hou et al. (2019), Figure 13, that the Black-Scholes implied volatilities for the Bitcoin option prices simulated from a co-jump price-volatility model in Bandi and Renó (2016) exhibit implied volatility skew for different maturities.

In the sequel, we shall study the impacts of introducing the variance component in the specification for the pricing kernel under the Heston-Nandi model with normal innovations on the European call and put Bitcoin option prices and their implied volatilities. For this, we vary the parameter ξ. When $\xi = 0$, the variance-dependent pricing kernel reduces to the pricing kernel selected by the conditional Esscher transform. When $\xi > 0$, the variance premium is negative. In Christoffersen et al. (2013), Table 5 on Page 1992, the value of ξ was used such that $\frac{1}{1-2\alpha_1\xi} = 1.2836$, where their estimate for α_1 was 3.364×10^{-6}. Here we assume that ξ takes values 0, 100, 200 and 300. The respective values of $\frac{1}{1-2\alpha_1\xi}$ are 1, 1.10, 1.22 and 1.37, where the estimate for α_1 here is $4.524075e-04$ (see Table A.1). Note also that the variance premium parameter in Table 4 of Badescu et al. (2017) is negative, which is equal to -0.328.

Tables A.9-A.14 below present the prices for the European call and put Bitcoin options with different strikes and maturities from the estimated Heston-Nandi GARCH model, where the variance-dependent pricing kernel is used with $\xi = 100, 200, 300$.

∼ Tables A.9-A.14 about here ∼

Again, the overall patterns of the prices for the European call and put Bitcoin options evaluated from the Heston-Nandi GARCH model using the variance-dependent pricing kernel with $\xi = 100, 200, 300$ appear to be consistent with the respective prices from the SETAR-Heston-Nandi-GARCH model and the Heston-Nandi GARCH model presented in Tables A.3-A.4 and Tables A.5-A.6, respectively, when the conditional Esscher transform is used. Specifically, the prices of the European call and put Bitcoin options increase when the maturity increases. The prices of the European call Bitcoin option decrease when the strike price increases. The prices of the European put Bitcoin options increase as the strike price increases.

Figures B.4-B.5 below depict the plots of the European call and put Bitcoin option prices, respectively, against maturity under the Heston-Nandi GARCH model using the variance-dependent pricing kernel for ITM, ATM and OTM options where $\xi = 0, 100, 200, 300$.

∼ Figures B.4-B.5 about here ∼

From Figure B.4, it can be seen that the European call Bitcoin option prices evaluated from the Heston-Nandi GARCH model using the variance-dependent pricing kernel increase when the parameter ξ increases from 0 to 300. Furthermore, from Figure B.5, it can be seen From Figure B.5, it can be observed that the European put Bitcoin option prices evaluated from the Heston-Nandi GARCH model using the variance-dependent pricing kernel also increase when the parameter ξ increases from 0 to 300. These results indicate that the incorporation of the variance premium in the pricing kernel increases both the European call and put Bitcoin option prices.

Figure B.6 below depicts the plots of the Black-Scholes implied volatilities for European Bitcoin call options against moneyness from the Heston-Nandi GARCH model using the variance-dependent pricing kernel for three different maturities, where $\xi = 0, 100, 200, 300$.

∼ Figure B.6 about here ∼

From Figure B.6, it can be seen that the Black-Scholes implied volatilities from the Heston-Nandi GARCH model using the variance-dependent pricing kernel increase when ξ increases from 0 to 300. This is consistent with the results depicted in Figure B.5.

4. Conclusion

A SETAR-GARCH model with the Heston-Nandi GARCH component was used to price European Bitcoin call and put options with a view to incorporating conditional heteroscedasticity and regime switching. The conditional Esscher transform and the variance-dependent pricing kernel were applied to specify a pricing kernel or an equivalent martingale measure. Monte Carlo simulations were employed to compute the Bitcoin options prices under the SETAR-Heston-Nandi-GARCH model, while the analytical pricing formula based on the characteristic function in Heston and Nandi (2000) was used to compute the Bitcoin options prices under the Heston-Nandi GARCH model. Using real data on Bitcoin exchange rates against the U.S. dollar, say the CoinDesk Index data, the impacts of conditional heteroscedasticity and regime switching on the Bitcoin options prices were examined. It was found that the impact of conditional heteroscedasticity on Bitcoin options prices appeared to be quite significant. However, the impact of self-exciting threshold regime switching effect on the Bitcoin options prices seemed to be marginal. Furthermore, the numerical results may indicate that the Black-Scholes implied volatilities for European Bitcoin call prices from the SETAR-Heston-Nandi-GARCH model and the Heston-Nandi GARCH model slightly exhibited implied volatility skew for short-maturity options. It was also found that the incorporation of the variance premium using the variance-dependent pricing kernel increases the European call and put prices evaluated from the Heston-Nandi GARCH model as well as the respective Black-Scholes implied volatilities.

An understanding of the behavior of Bitcoin option prices and their respective Black-Scholes implied volatilities under the one-price theory based on the absence of arbitrage and the law of one price may throw light on the potential development of a two-price theory for valuing Bitcoin or cryptocurrency options and the exploration of its potential implications for market efficiency of Bitcoin or cryptocurrency options and arbitrages in these markets. For future research, one may explore the possibility of using the two-price conic finance theory in, for example, Madan and Cherny (2010) and Madan (2012) to develop a two-price theory for Bitcoin or cryptocurrency options under the SETAR-GARCH and GARCH models. One may also investigate the market efficiency of Bitcoin or cryptocurrency option markets under the SETAR-GARCH and GARCH models using the method in Madan et al. (2017), which was based on the two-price conic finance theory. The current option pricing theory under SETAR-GARCH and GARCH models based on the law of one price may provide some insights for this endeavour. Specifically, according to Madan et al. (2017), the law of one price from market efficiency may be related to the largest market cone with the half space, while the other extreme of the smallest market cone is given by the set of positive random variables which may correspond to arbitrage. There is a spectrum of market cones in between which may give rise to two prices for Bitcoin or cryptocurrency options which are not bilateral. The study of market efficiency of Bitcoin or cryptocurrency options may provide some insights into understanding to what extent arbitrage opportunities in real Bitcoin markets may impact the risk-neutral valuation of Bitcoin options. Besides the use of the two-price conic finance theory, it may also be interesting to explore the use of market microstructure theory (see, for example, the monograph by O'Hara (1998)) to develop a two-price theory for Bitcoin or cryptocurrency options and to study market efficiency of Bitcoin or cryptocurrency options. For the investigation of market efficiency of Bitcoin or cryptocurrency options, some tests or methods in, for example, Black and Scholes (1972), Chiras and Manaster (1978) and Jarrow (2013), may also be considered.

Another potential topic for future research may be to study the hedging of Bitcoin options under the SETAR-GARCH model and the GARCH model. The paper by Siu et al. (2014), where the Delta and Delta-Gamma hedges were applied to hedge European-type crude oil options under the GARCH models with normal and non-normal innovations and their empirical performances were studied, may be relevant for this endeavor.

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Appendix: Derivations

Heuristic Derivation of the Unconditional Risk-Neutral Variance Under the SETAR-GARCH Model

Recall from Eq. (2.9) that

$$h_t = \alpha_0 + \alpha_1(\epsilon^*_{t-1} - (\gamma + \lambda(t-1))\sqrt{h_{t-1}})^2 + \beta h_{t-1}, \tag{2.25}$$

where $\lambda(t-1) = \lambda_1 I_{\{R_{t-2} \geq r\}} + \lambda_2 I_{\{R_{t-2} < r\}}$.
Let $\gamma^*_t := (\gamma + \lambda_1) I_{\{R_{t-1} \geq r\}} + (\gamma + \lambda_2) I_{\{R_{t-1} < r\}}$. Then

$$h_t = \alpha_0 + \alpha_1(\epsilon^*_{t-1})^2 - 2\alpha_1 \epsilon^*_{t-1}\gamma^*_{t-1}\sqrt{h_{t-1}} + \alpha_1(\gamma^*_{t-1})^2 h_{t-1} + \beta h_{t-1}. \tag{2.26}$$

Assume covariance stationarity holds under the risk-neutral probability measure $\mathbb{P}^\theta$ selected by the conditional Esscher transform. Then $\mathrm{E}^\theta[h_t] = \mathrm{E}^\theta[h_{t-1}]$, where E^θ is the expectation under $\mathbb{P}^\theta$.
Taking expectation on both sides of Eq. (2.26) with respect to $\mathbb{P}^\theta$ gives:

$$\begin{aligned}\mathrm{E}^\theta[h_t] &= \alpha_0 + \alpha_1 \mathrm{E}^\theta[(\epsilon^*_{t-1})^2] - 2\alpha_1 \mathrm{E}^\theta[\epsilon^*_{t-1}\gamma^*_{t-1}\sqrt{h_{t-1}}] + \mathrm{E}^\theta[(\gamma^*_{t-1})^2 h_{t-1}] \\ &\quad + \beta \mathrm{E}^\theta[h_{t-1}]. \end{aligned} \tag{2.27}$$

Since $\epsilon^*_t|\mathcal{F}_{t-1} \sim N(0,1)$ under $\mathbb{P}^\theta$,

$$\begin{aligned}\mathrm{E}^\theta[(\epsilon^*_{t-1})^2] &= \mathrm{E}^\theta[\mathrm{E}^\theta[(\epsilon^*_{t-1})^2|\mathcal{F}_{t-2}]] \\ &= \mathrm{E}^\theta[1] = 1. \end{aligned} \tag{2.28}$$

Also, since γ^*_{t-1} and h_{t-1} are $\mathcal{F}_{t-2}$-measurable,

$$\begin{aligned}\mathrm{E}^\theta[\epsilon^*_{t-1}\gamma^*_{t-1}\sqrt{h_{t-1}}] &= \mathrm{E}^\theta[\mathrm{E}^\theta[\epsilon^*_{t-1}\gamma^*_{t-1}\sqrt{h_{t-1}}|\mathcal{F}_{t-2}]] \\ &= \mathrm{E}^\theta[\gamma^*_{t-1}\sqrt{h_{t-1}}\mathrm{E}^\theta[\epsilon^*_{t-1}|\mathcal{F}_{t-2}]] = 0. \end{aligned} \tag{2.29}$$

As in Wu (2011), Page 8, let S_t be the state of the regime at time t such that $S_t = 1$ if and only if $R_{t-1} \geq r$ and $S_t = 0$ if and only if $R_{t-1} < r$. Let $\pi := \mathbb{P}^\theta(S_t = 1) = \mathbb{P}^\theta(R_{t-1} \geq r)$. It is supposed that π does not depend on time t. Consequently,

$$\pi = \mathbb{P}^\theta(S_t = 1) = \mathbb{P}^\theta(S_{t-1} = 1) = \mathbb{P}^\theta(R_{t-2} \geq r). \tag{2.30}$$

Since $\gamma^*_{t-1} := (\gamma + \lambda_1) I_{\{R_{t-2} \geq r\}} + (\gamma + \lambda_2) I_{\{R_{t-2} < r\}}$, by Eq. (2.30),

$$\begin{aligned} & \mathbb{E}^\theta[(\gamma^*_{t-1})^2 h_{t-1}] \\ = \;& \mathbb{E}^\theta[(\gamma^*_{t-1})^2 h_{t-1}|S_{t-1} = 1]\mathbb{P}^\theta(S_{t-1} = 1) + \mathbb{E}^\theta[(\gamma^*_{t-1})^2 h_{t-1}|S_{t-1} = 0]\mathbb{P}^\theta(S_{t-1} = 0) \\ = \;& \mathbb{E}^\theta[(\gamma^*_{t-1})^2 h_{t-1}|R_{t-2} \geq r]\mathbb{P}^\theta(S_{t-1} = 1) + \mathbb{E}^\theta[(\gamma^*_{t-1})^2 h_{t-1}|R_{t-2} < r]\mathbb{P}^\theta(S_{t-1} = 0) \\ = \;& (\gamma + \lambda_1)^2 \mathbb{E}^\theta[h_{t-1}|R_{t-2} \geq r]\mathbb{P}^\theta(S_{t-1} = 1) + (\gamma + \lambda_2)^2 \mathbb{E}^\theta[h_{t-1}|R_{t-2} < r]\mathbb{P}^\theta(S_{t-1} = 0) \\ = \;& (\gamma + \lambda_1)^2 \mathbb{E}^\theta[h_{t-1}]\pi + (\gamma + \lambda_2)^2 \mathbb{E}^\theta[h_{t-1}](1 - \pi). \end{aligned} \tag{2.31}$$

Using Eq. (2.27), Eq. (2.28), Eq. (2.29) and Eq. (2.31),

$$\begin{aligned} \mathrm{E}^{\theta}[h_t] &= \alpha_0 + \alpha_1 + \alpha_1(\gamma + \lambda_1)^2 \mathrm{E}^{\theta}[h_{t-1}]\pi + \alpha_1(\gamma + \lambda_2)^2 \mathrm{E}^{\theta}[h_{t-1}](1 - \pi) \\ &\quad + \beta \mathrm{E}^{\theta}[h_{t-1}]. \end{aligned} \tag{2.32}$$

Since $\mathrm{E}^{\theta}[h_t] = \mathrm{E}^{\theta}[h_{t-1}]$,

$$\mathrm{E}^{\theta}[h_t] = \frac{\alpha_0 + \alpha_1}{1 - \alpha_1(\gamma + \lambda_1)^2\pi - \alpha_1(\gamma + \lambda_2)^2(1 - \pi) - \beta}. \tag{2.33}$$

Table A.1. Estimation Results for the SETAR-GARCH Model with the Heston-Nandi GARCH(1,1) component and the Heston-Nandi GARCH(1,1) Model

Model	λ_1	λ_2	λ	α_0	α_1	β	γ	r	Neg. Log-Likelihood
SETAR-Heston-Nandi-GARCH	9.999990e-01	2.925598e-01	-	5.436851e-05	4.524075e-04	8.238616e-01	1.000000e-06	-0.007873053	-4797.563
	(5.362202e+00)	(3.484125e-01)	-	(8.013229e-06)	(4.332673e-05)	(1.241648e-02)	(5.463017e+00)	-	-
Heston-Nandi GARCH	-	-	9.999990e-01	5.435065e-05	4.520402e-04	8.239117e-01	1.000000e-06	-	-4796.812
	-	-	(3.490085e-01)	(8.012520e-06)	(4.337594e-05)	(1.242152e-02)	(1.336980e+00)	-	-

Table A.2. Estimated Unconditional Risk-Neutral Variances Under the SETAR-GARCH Model with the Heston-Nandi GARCH(1,1) component and the Heston-Nandi GARCH(1,1) Model

SETAR-Heston-Nandi-GARCH	Heston-Nandi-GARCH ($\xi = 0$)	Heston-Nandi-GARCH ($\xi = 100$)	Heston-Nandi-GARCH ($\xi = 200$)	Heston-Nandi-GARCH ($\xi = 300$)
0.00288252	0.002883179	0.003452844	0.004218382	0.005282524

Table A.3. Monte Carlo Estimates and Standard Errors for European Call Bitcoin Prices Under the SETAR-Heston-Nandi-GARCH Model using the Conditional Esscher Transform

$K\backslash T$	30	60	90	120	150	180	210	240	270	300	330	360
$S_0(1-0.2)$	1,775.355	2,007.571	2,252.902	2,409.786	2,688.493	2,734.182	2,892.654	3,080.434	3,354.967	3,433.384	3,379.243	3,615.438
	(20.292)	(28.053)	(36.136)	(41.272)	(49.301)	(52.549)	(58.451)	(67.711)	(82.397)	(83.668)	(82.789)	(98.378)
$S_0(1-0.16)$	1,531.194	1,837.708	2,064.020	2,257.086	2,450.952	2,610.743	2,879.940	2,949.308	2,981.599	3,169.770	3,247.762	3,401.372
	(19.110)	(27.179)	(34.971)	(40.185)	(45.931)	(52.297)	(61.602)	(66.459)	(71.084)	(81.269)	(88.080)	(94.504)
$S_0(1-0.12)$	1,351.935	1,674.627	1,889.429	2,037.284	2,283.444	2,475.507	2,599.164	2,872.102	2,967.728	3,029.233	3,273.126	3,420.207
	(18.564)	(26.466)	(32.922)	(37.923)	(45.267)	(52.896)	(58.090)	(71.201)	(71.365)	(79.950)	(94.036)	(100.064)
$S_0(1-0.08)$	1,179.341	1,519.644	1,818.669	2,093.862	2,164.976	2,415.010	2,398.047	2,710.025	2,797.133	2,989.754	3,110.197	3,181.840
	(17.413)	(25.555)	(32.422)	(40.304)	(44.564)	(53.987)	(57.207)	(64.752)	(75.022)	(78.696)	(80.867)	(92.824)
$S_0(1-0.04)$	976.808	1,374.062	1,604.082	1,870.237	2,101.064	2,148.627	2,374.408	2,471.503	2,719.686	2,932.773	3,073.229	2,998.501
	(16.350)	(23.787)	(31.044)	(39.069)	(45.854)	(49.554)	(56.673)	(63.608)	(70.757)	(94.404)	(83.208)	(89.197)
S_0	883.664	1,229.790	1,497.763	1,742.943	1,978.791	2,161.902	2,368.258	2,401.412	2,549.089	2,858.824	2,921.260	2,978.510
	(16.071)	(23.763)	(29.675)	(37.328)	(45.264)	(49.816)	(57.665)	(61.404)	(67.573)	(84.094)	(97.614)	(86.617)
$S_0(1+0.04)$	727.107	1,087.424	1,342.942	1,657.931	1,882.998	1,944.939	2,155.584	2,230.835	2,596.486	2,618.275	2,737.856	2,952.849
	(14.204)	(23.647)	(28.865)	(37.991)	(46.001)	(47.141)	(56.501)	(57.119)	(69.003)	(71.229)	(77.785)	(90.002)
$S_0(1+0.08)$	637.839	1,028.883	1,285.813	1,541.086	1,674.936	1,933.937	2,063.187	2,240.626	2,420.183	2,564.298	2,737.118	2,924.429
	(13.895)	(22.501)	(28.670)	(37.483)	(40.956)	(49.653)	(53.384)	(62.075)	(69.210)	(73.818)	(82.601)	(97.849)
$S_0(1+0.12)$	533.124	893.816	1,211.985	1,442.946	1,643.151	1,804.524	1,996.334	2,203.972	2,209.116	2,396.262	2,534.170	2,746.046
	(12.755)	(20.814)	(28.407)	(34.444)	(43.140)	(46.238)	(53.578)	(60.189)	(66.593)	(75.655)	(77.521)	(87.693)
$S_0(1+0.16)$	474.022	805.806	1,110.728	1,324.295	1,536.691	1,752.575	1,943.573	2,157.277	2,166.607	2,302.891	2,586.026	2,759.479
	(12.202)	(20.203)	(27.395)	(33.787)	(39.657)	(45.844)	(54.800)	(64.922)	(64.016)	(82.386)	(80.623)	(90.739)
$S_0(1+0.2)$	383.276	746.554	987.078	1,257.059	1,499.606	1,654.990	1,926.790	2,062.718	2,213.959	2,352.979	2,575.966	2,674.506
	(11.058)	(19.391)	(26.435)	(32.385)	(43.496)	(45.736)	(55.930)	(61.702)	(70.319)	(72.201)	(83.278)	(83.685)

Table A.4. Monte Carlo Estimates and Standard Errors for European Put Bitcoin Prices Under the the SETAR-Heston-Nandi-GARCH Model using the Conditional Esscher Transform

$K\backslash T$	30	60	90	120	150	180	210	240	270	300	330	360
$S_0(1-0.2)$	242.322	509.654	715.159	900.056	1,076.732	1,217.636	1,330.430	1,467.324	1,553.270	1,686.461	1,822.528	1,864.207
	(5.587)	(8.779)	(10.992)	(12.778)	(14.276)	(15.212)	(16.088)	(16.977)	(17.548)	(18.352)	(19.021)	(19.312)
$S_0(1-0.16)$	335.114	653.197	866.310	1,046.315	1,218.595	1,365.820	1,496.281	1,625.885	1,769.741	1,885.681	1,994.893	2,081.473
	(6.622)	(10.191)	(12.258)	(13.937)	(15.328)	(16.280)	(17.317)	(18.042)	(18.819)	(19.436)	(19.940)	(20.509)
$S_0(1-0.12)$	432.074	736.147	993.793	1,217.890	1,388.302	1,554.120	1,692.774	1,795.207	1,924.423	2,086.443	2,160.225	2,280.980
	(7.665)	(10.985)	(13.301)	(15.127)	(16.574)	(17.697)	(18.508)	(19.144)	(20.041)	(20.611)	(20.994)	(21.759)
$S_0(1-0.08)$	553.080	896.722	1,113.458	1,347.979	1,530.795	1,674.370	1,879.581	1,960.122	2,136.840	2,276.527	2,351.739	2,486.420
	(8.754)	(12.212)	(14.352)	(16.193)	(17.398)	(18.489)	(19.691)	(20.256)	(21.088)	(21.829)	(22.231)	(22.936)
$S_0(1-0.04)$	701.863	1,047.612	1,299.851	1,538.882	1,659.611	1,889.245	2,043.332	2,166.294	2,272.338	2,411.336	2,554.543	2,703.424
	(9.767)	(13.298)	(15.457)	(17.271)	(18.273)	(19.612)	(20.772)	(21.408)	(22.121)	(22.864)	(23.557)	(24.018)
S_0	845.260	1,216.142	1,453.440	1,690.067	1,905.749	2,059.857	2,204.311	2,348.397	2,483.791	2,647.101	2,760.970	2,847.657
	(10.905)	(14.466)	(16.376)	(18.252)	(19.805)	(20.821)	(21.762)	(22.604)	(23.301)	(24.026)	(24.424)	(24.995)
$S_0(1+0.04)$	1,028.521	1,386.767	1,682.276	1,878.638	2,120.955	2,305.658	2,430.529	2,599.072	2,696.856	2,828.396	2,948.070	3,064.576
	(11.925)	(15.311)	(17.801)	(19.461)	(20.956)	(21.917)	(22.584)	(23.880)	(24.499)	(25.001)	(25.515)	(25.996)
$S_0(1+0.08)$	1,210.750	1,576.192	1,807.974	2,087.903	2,317.283	2,474.967	2,610.263	2,788.785	2,899.313	3,034.746	3,177.499	3,292.604
	(12.977)	(16.417)	(18.508)	(20.461)	(21.937)	(23.142)	(23.993)	(24.925)	(25.483)	(26.168)	(26.737)	(27.246)
$S_0(1+0.12)$	1,432.678	1,815.759	2,046.224	2,299.659	2,521.792	2,667.999	2,836.741	2,965.406	3,186.343	3,285.849	3,425.277	3,451.633
	(13.918)	(17.539)	(19.793)	(21.556)	(23.013)	(23.948)	(25.148)	(25.683)	(26.432)	(27.085)	(27.665)	(28.129)
$S_0(1+0.16)$	1,638.242	1,962.912	2,266.280	2,495.471	2,677.289	2,882.075	3,038.839	3,190.371	3,337.865	3,486.887	3,574.704	3,723.873
	(14.746)	(18.163)	(20.756)	(22.371)	(23.850)	(25.176)	(25.960)	(26.848)	(27.347)	(28.078)	(28.454)	(29.229)
$S_0(1+0.2)$	1,860.739	2,177.520	2,469.463	2,696.875	2,902.711	3,145.016	3,256.312	3,425.087	3,591.360	3,714.767	3,799.027	3,904.118
	(15.498)	(19.333)	(21.748)	(23.557)	(24.970)	(26.177)	(27.049)	(28.073)	(28.410)	(29.101)	(29.656)	(30.161)

Table A.5. European Call Bitcoin Prices Under the Heston-Nandi GARCH Model using the Conditional Esscher Transform or the Variance-Dependent Pricing Kernel with $\xi = 0$

$K \backslash T$	30	60	90	120	150	180	210	240	270	300	330	360
$S_0(1-0.2)$	1,762.183	2,027.235	2,248.024	2,439.124	2,609.058	2,762.966	2,904.189	3,035.038	3,157.185	3,271.889	3,380.125	3,482.666
$S_0(1-0.16)$	1,548.733	1,843.229	2,080.290	2,282.636	2,461.220	2,622.202	2,769.442	2,905.546	3,032.373	3,151.306	3,263.406	3,369.510
$S_0(1-0.12)$	1,352.479	1,672.537	1,923.634	2,135.770	2,321.960	2,489.221	2,641.843	2,782.681	2,913.750	3,036.535	3,152.173	3,261.552
$S_0(1-0.08)$	1,173.986	1,514.889	1,777.652	1,998.097	2,190.857	2,363.616	2,521.004	2,666.071	2,800.963	2,927.242	3,046.104	3,158.483
$S_0(1-0.04)$	1,013.351	1,369.877	1,641.887	1,869.174	2,067.490	2,244.991	2,406.553	2,555.371	2,693.679	2,823.108	2,944.897	3,060.014
S_0	870.231	1,236.977	1,515.845	1,748.545	1,951.445	2,132.964	2,298.134	2,450.243	2,591.585	2,723.837	2,848.271	2,965.880
$S_0(1+0.04)$	743.909	1,115.582	1,399.010	1,635.769	1,842.316	2,027.165	2,195.406	2,350.370	2,494.385	2,629.153	2,755.966	2,875.832
$S_0(1+0.08)$	633.370	1,005.028	1,290.857	1,530.389	1,739.711	1,927.242	2,098.047	2,255.456	2,401.802	2,538.796	2,667.736	2,789.638
$S_0(1+0.12)$	537.393	904.615	1,190.857	1,431.971	1,643.251	1,832.857	2,005.751	2,165.221	2,313.577	2,452.522	2,583.352	2,707.084
$S_0(1+0.16)$	454.634	813.630	1,098.489	1,340.092	1,552.573	1,743.689	1,918.230	2,079.399	2,229.464	2,370.105	2,502.603	2,627.968
$S_0(1+0.2)$	383.705	731.360	1,013.245	1,254.344	1,467.331	1,659.435	1,835.210	1,997.743	2,149.236	2,291.331	2,425.289	2,552.105

Table A.6. European Put Bitcoin Prices Under the Heston-Nandi GARCH Model using the Conditional Esscher Transform or the Variance-Dependent Pricing Kernel with $\xi = 0$

$K \backslash T$	30	60	90	120	150	180	210	240	270	300	330	360
$S_0(1-0.2)$	254.996	510.635	722.025	903.742	1,064.306	1,208.859	1,340.743	1,462.267	1,575.103	1,680.512	1,779.466	1,872.741
$S_0(1-0.16)$	340.627	625.239	852.431	1,044.924	1,213.670	1,364.830	1,502.263	1,628.575	1,745.626	1,854.799	1,957.154	2,053.528
$S_0(1-0.12)$	443.453	753.156	993.914	1,195.727	1,371.612	1,528.583	1,670.931	1,801.511	1,922.339	2,034.899	2,140.327	2,239.514
$S_0(1-0.08)$	564.040	894.118	1,146.072	1,355.725	1,537.711	1,699.712	1,846.359	1,980.703	2,104.887	2,220.476	2,328.665	2,430.387
$S_0(1-0.04)$	702.485	1,047.715	1,308.446	1,524.472	1,711.545	1,877.821	2,028.176	2,165.804	2,292.939	2,411.213	2,521.864	2,625.862
S_0	858.446	1,213.424	1,480.544	1,701.514	1,892.702	2,062.528	2,216.023	2,356.476	2,486.180	2,606.813	2,719.646	2,825.671
$S_0(1+0.04)$	1,031.204	1,390.639	1,661.848	1,886.408	2,080.775	2,253.464	2,409.562	2,552.404	2,684.316	2,806.999	2,921.747	3,029.566
$S_0(1+0.08)$	1,219.745	1,578.694	1,851.835	2,078.699	2,275.372	2,450.275	2,608.471	2,753.291	2,887.068	3,011.512	3,127.923	3,237.316
$S_0(1+0.12)$	1,422.848	1,776.891	2,049.974	2,277.951	2,476.114	2,652.624	2,812.442	2,958.857	3,094.178	3,220.109	3,337.946	3,448.705
$S_0(1+0.16)$	1,639.169	1,984.516	2,255.746	2,483.743	2,682.638	2,860.190	3,021.189	3,168.836	3,305.401	3,432.563	3,551.603	3,663.532
$S_0(1+0.2)$	1,867.321	2,200.855	2,468.642	2,695.665	2,894.598	3,072.670	3,234.436	3,382.981	3,520.508	3,648.660	3,768.696	3,881.612

Table A.7. European Bitcoin Call Prices from the Black-Scholes-Merton Model

$K \backslash T$	30	60	90	120	150	180	210	240	270	300	330	360
$S_0(1-0.2)$	1,810.678	2,108.764	2,351.666	2,559.943	2,744.092	2,910.171	3,062.033	3,202.309	3,332.892	3,455.197	3,570.318	3,679.121
$S_0(1-0.16)$	1,607.706	1,933.639	2,192.051	2,411.195	2,603.795	2,776.842	2,934.666	3,080.175	3,215.434	3,341.976	3,460.977	3,573.363
$S_0(1-0.12)$	1,421.144	1,770.856	2,042.629	2,271.265	2,471.337	2,650.608	2,813.802	2,964.057	3,103.584	3,234.012	3,356.587	3,472.287
$S_0(1-0.08)$	1,250.987	1,619.993	1,902.939	2,139.707	2,346.296	2,531.070	2,699.066	2,853.603	2,997.006	3,130.987	3,256.847	3,375.604
$S_0(1-0.04)$	1,096.911	1,480.548	1,772.502	2,016.077	2,228.262	2,417.851	2,590.106	2,748.480	2,895.391	3,032.608	3,161.477	3,283.050
S_0	958.327	1,351.963	1,650.825	1,899.937	2,116.841	2,310.588	2,486.586	2,648.380	2,798.446	2,938.600	3,070.219	3,194.380
$S_0(1+0.04)$	834.448	1,233.642	1,537.418	1,790.862	2,011.653	2,208.938	2,388.193	2,553.010	2,705.902	2,848.710	2,982.834	3,109.368
$S_0(1+0.08)$	724.343	1,124.969	1,431.792	1,688.440	1,912.338	2,112.579	2,294.633	2,462.100	2,617.505	2,762.702	2,899.101	3,027.806
$S_0(1+0.12)$	626.989	1,025.322	1,333.473	1,592.275	1,818.551	2,021.203	2,205.627	2,375.395	2,533.021	2,680.357	2,818.814	2,949.498
$S_0(1+0.16)$	541.317	934.085	1,241.999	1,501.988	1,729.968	1,934.523	2,120.916	2,292.658	2,452.231	2,601.471	2,741.781	2,874.267
$S_0(1+0.2)$	466.253	850.655	1,156.926	1,417.220	1,646.280	1,852.266	2,040.255	2,213.665	2,374.928	2,525.852	2,667.826	2,801.943

Table A.8. European Bitcoin Put Prices from the Black-Scholes-Merton Model

$K \backslash T$	30	60	90	120	150	180	210	240	270	300	330	360
$S_0(1-0.2)$	303.491	592.163	825.667	1,024.560	1,199.340	1,356.065	1,498.587	1,629.538	1,750.810	1,863.819	1,969.659	2,069.196
$S_0(1-0.16)$	399.600	715.649	964.192	1,173.483	1,356.245	1,519.470	1,667.487	1,803.204	1,928.688	2,045.469	2,154.725	2,257.381
$S_0(1-0.12)$	512.118	851.475	1,112.909	1,331.223	1,520.989	1,689.969	1,842.890	1,982.887	2,112.173	2,232.376	2,344.742	2,450.248
$S_0(1-0.08)$	641.041	999.221	1,271.359	1,497.335	1,693.150	1,867.166	2,024.422	2,168.234	2,300.931	2,424.222	2,539.408	2,647.508
$S_0(1-0.04)$	786.045	1,158.386	1,439.061	1,671.375	1,872.318	2,050.681	2,211.728	2,358.912	2,494.650	2,620.712	2,738.444	2,848.898
S_0	946.541	1,328.410	1,615.524	1,852.906	2,058.098	2,240.152	2,404.475	2,554.613	2,693.041	2,821.575	2,941.593	3,054.171
$S_0(1+0.04)$	1,121.743	1,508.699	1,800.256	2,041.501	2,250.112	2,435.237	2,602.350	2,755.044	2,895.832	3,026.556	3,148.615	3,263.103
$S_0(1+0.08)$	1,310.718	1,698.635	1,992.770	2,236.749	2,447.999	2,635.611	2,805.057	2,959.935	3,102.771	3,235.419	3,359.288	3,475.483
$S_0(1+0.12)$	1,512.444	1,897.598	2,192.591	2,438.255	2,651.414	2,840.970	3,012.318	3,169.031	3,313.623	3,447.944	3,573.407	3,691.119
$S_0(1+0.16)$	1,725.853	2,104.971	2,399.256	2,645.639	2,860.033	3,051.024	3,223.874	3,382.094	3,528.168	3,663.929	3,790.782	3,909.831
$S_0(1+0.2)$	1,949.869	2,320.150	2,612.323	2,858.541	3,073.547	3,265.501	3,439.481	3,598.903	3,746.200	3,883.181	4,011.233	4,131.451

Table A.9. European Call Bitcoin Prices Under the Heston-Nandi GARCH Model using the Variance-Dependent Pricing Kernel with $\xi = 100$

$K \backslash T$	30	60	90	120	150	180	210	240	270	300	330	360
$S_0(1-0.2)$	1,818.487	2,117.243	2,361.945	2,571.997	2,757.749	2,925.253	3,078.384	3,219.795	3,351.397	3,474.623	3,590.577	3,700.136
$S_0(1-0.16)$	1,612.497	1,940.264	2,201.071	2,422.340	2,616.775	2,791.417	2,950.642	3,097.392	3,233.758	3,361.295	3,481.194	3,594.392
$S_0(1-0.12)$	1,422.727	1,775.595	2,050.360	2,281.465	2,483.602	2,664.636	2,829.361	2,980.962	3,121.684	3,253.181	3,376.718	3,493.286
$S_0(1-0.08)$	1,249.456	1,622.898	1,909.394	2,148.953	2,357.825	2,544.524	2,714.177	2,870.162	3,014.847	3,149.970	3,276.855	3,396.536
$S_0(1-0.04)$	1,092.603	1,481.740	1,777.723	2,024.379	2,239.049	2,430.714	2,604.746	2,764.668	2,912.943	3,051.374	3,181.331	3,303.884
S_0	951.755	1,351.609	1,654.883	1,907.323	2,126.891	2,322.854	2,500.742	2,664.178	2,815.689	2,957.126	3,089.894	3,215.090
$S_0(1+0.04)$	826.224	1,231.944	1,540.398	1,797.371	2,020.982	2,220.608	2,401.859	2,568.405	2,722.818	2,866.976	3,002.309	3,129.930
$S_0(1+0.08)$	715.109	1,122.147	1,433.794	1,694.120	1,920.968	2,123.660	2,307.803	2,477.083	2,634.082	2,780.693	2,918.357	3,048.201
$S_0(1+0.12)$	617.361	1,021.602	1,334.604	1,597.182	1,826.511	2,031.709	2,218.307	2,389.963	2,549.250	2,698.061	2,837.837	2,969.710
$S_0(1+0.16)$	531.841	929.688	1,242.368	1,506.182	1,737.290	1,944.469	2,133.111	2,306.809	2,468.106	2,618.878	2,760.560	2,894.281
$S_0(1+0.2)$	457.380	845.790	1,156.643	1,420.762	1,652.999	1,861.673	2,051.975	2,227.403	2,390.447	2,542.957	2,686.351	2,821.750

Table A.10. European Put Bitcoin Prices Under the Heston-Nandi GARCH Model using the Variance-Dependent Pricing Kernel with $\xi = 100$

$K\backslash T$	30	60	90	120	150	180	210	240	270	300	330	360
$S_0(1-0.2)$	311.301	600.643	835.946	1,036.615	1,212.997	1,371.146	1,514.937	1,647.023	1,769.316	1,883.245	1,989.918	2,090.211
$S_0(1-0.16)$	404.391	722.273	973.212	1,184.627	1,369.225	1,534.045	1,683.463	1,820.421	1,947.012	2,064.788	2,174.941	2,278.410
$S_0(1-0.12)$	513.700	856.214	1,120.641	1,341.423	1,533.254	1,703.997	1,858.449	1,999.792	2,130.273	2,251.545	2,364.872	2,471.247
$S_0(1-0.08)$	639.510	1,002.127	1,277.814	1,506.581	1,704.679	1,880.620	2,039.532	2,184.793	2,318.771	2,443.204	2,559.416	2,668.441
$S_0(1-0.04)$	781.738	1,159.578	1,444.283	1,679.678	1,883.105	2,063.544	2,226.368	2,375.101	2,512.203	2,639.479	2,758.299	2,869.732
S_0	939.970	1,328.057	1,619.581	1,860.292	2,068.148	2,252.418	2,418.631	2,570.411	2,710.284	2,840.102	2,961.268	3,074.881
$S_0(1+0.04)$	1,113.519	1,507.001	1,803.236	2,048.010	2,259.441	2,446.907	2,616.016	2,770.439	2,912.748	3,044.823	3,168.089	3,283.664
$S_0(1+0.08)$	1,301.484	1,695.813	1,994.772	2,242.430	2,456.629	2,646.693	2,818.227	2,974.919	3,119.348	3,253.410	3,378.544	3,495.878
$S_0(1+0.12)$	1,502.816	1,893.878	2,193.721	2,443.162	2,659.374	2,851.476	3,024.998	3,183.599	3,329.852	3,465.648	3,592.431	3,711.331
$S_0(1+0.16)$	1,716.376	2,100.573	2,399.625	2,649.832	2,867.355	3,060.970	3,236.069	3,396.246	3,544.043	3,681.336	3,809.561	3,929.845
$S_0(1+0.2)$	1,940.995	2,315.285	2,612.040	2,862.082	3,080.266	3,274.908	3,451.200	3,612.641	3,761.719	3,900.286	4,029.758	4,151.257

Table A.11. European Call Bitcoin Prices Under the Heston-Nandi GARCH Model using the Variance-Dependent Pricing Kernel with $\xi = 200$

$K\backslash T$	30	60	90	120	150	180	210	240	270	300	330	360
$S_0(1-0.2)$	1,890.572	2,229.594	2,502.690	2,735.107	2,939.390	3,122.695	3,289.542	3,443.005	3,585.285	3,718.029	3,842.504	3,959.714
$S_0(1-0.16)$	1,692.724	2,060.300	2,349.389	2,593.050	2,806.101	2,996.652	3,169.707	3,328.621	3,475.775	3,612.933	3,741.448	3,862.384
$S_0(1-0.12)$	1,509.968	1,902.240	2,205.287	2,458.899	2,679.804	2,876.905	3,055.618	3,219.533	3,371.181	3,512.428	3,644.700	3,769.113
$S_0(1-0.08)$	1,342.351	1,755.032	2,069.966	2,332.252	2,560.117	2,763.100	2,946.942	3,115.427	3,271.208	3,416.236	3,551.996	3,679.651
$S_0(1-0.04)$	1,189.652	1,618.234	1,942.996	2,212.710	2,446.677	2,654.899	2,843.367	3,016.011	3,175.582	3,324.098	3,463.093	3,593.766
S_0	1,051.414	1,491.358	1,823.944	2,099.889	2,339.135	2,551.986	2,744.600	2,921.014	3,084.051	3,235.778	3,377.768	3,511.249
$S_0(1+0.04)$	926.990	1,373.890	1,712.381	1,993.416	2,237.160	2,454.060	2,650.367	2,830.183	2,996.378	3,151.056	3,295.814	3,431.905
$S_0(1+0.08)$	815.587	1,265.299	1,607.885	1,892.932	2,140.438	2,360.839	2,560.412	2,743.284	2,912.348	3,069.728	3,217.041	3,355.554
$S_0(1+0.12)$	716.312	1,165.047	1,510.043	1,798.096	2,048.671	2,272.060	2,474.496	2,660.095	2,831.755	2,991.606	3,141.273	3,282.030
$S_0(1+0.16)$	628.215	1,072.601	1,418.457	1,708.579	1,961.578	2,187.474	2,392.394	2,580.414	2,754.413	2,916.515	3,068.345	3,211.180
$S_0(1+0.2)$	550.322	987.439	1,332.746	1,624.073	1,878.894	2,106.846	2,313.897	2,504.049	2,680.145	2,844.292	2,998.107	3,142.862

Table A.12. European Put Bitcoin Prices Under the Heston-Nandi GARCH Model using the Variance-Dependent Pricing Kernel with $\xi = 200$

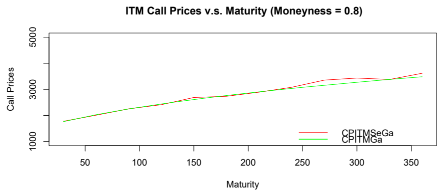
$K\backslash T$	30	60	90	120	150	180	210	240	270	300	330	360
$S_0(1-0.2)$	383.386	712.994	976.691	1,199.724	1,394.638	1,568.588	1,726.096	1,870.233	2,003.203	2,126.651	2,241.845	2,349.788
$S_0(1-0.16)$	484.618	842.310	1,121.530	1,355.337	1,558.551	1,739.279	1,902.528	2,051.651	2,189.029	2,316.426	2,435.195	2,546.402
$S_0(1-0.12)$	600.942	982.859	1,275.567	1,518.857	1,729.456	1,916.267	2,084.706	2,238.363	2,379.770	2,510.792	2,632.854	2,747.075
$S_0(1-0.08)$	732.405	1,134.261	1,438.386	1,689.880	1,906.971	2,099.196	2,272.297	2,430.058	2,575.132	2,709.470	2,834.557	2,951.555
$S_0(1-0.04)$	878.786	1,296.072	1,609.555	1,868.008	2,090.733	2,287.730	2,464.989	2,626.443	2,774.842	2,912.203	3,040.060	3,159.614
S_0	1,039.629	1,467.805	1,788.643	2,052.858	2,280.392	2,481.550	2,662.489	2,827.247	2,978.646	3,118.754	3,249.142	3,371.040
$S_0(1+0.04)$	1,214.285	1,648.947	1,975.220	2,244.055	2,475.619	2,680.358	2,864.523	3,032.217	3,186.309	3,328.902	3,461.595	3,585.639
$S_0(1+0.08)$	1,401.962	1,838.965	2,168.863	2,441.242	2,676.099	2,883.872	3,070.836	3,241.119	3,397.614	3,542.445	3,677.229	3,803.231
$S_0(1+0.12)$	1,601.767	2,037.323	2,369.160	2,644.076	2,881.534	3,091.826	3,281.187	3,453.731	3,612.357	3,759.193	3,895.867	4,023.651
$S_0(1+0.16)$	1,812.750	2,243.487	2,575.715	2,852.230	3,091.643	3,303.975	3,495.352	3,669.851	3,830.350	3,978.973	4,117.346	4,246.744
$S_0(1+0.2)$	2,033.938	2,456.933	2,788.143	3,065.393	3,306.161	3,520.081	3,713.122	3,889.287	4,051.417	4,201.621	4,341.514	4,472.370

Table A.13. European Call Bitcoin Prices Under the Heston-Nandi GARCH Model using the Variance-Dependent Pricing Kernel with $\xi = 300$

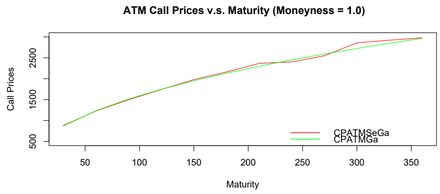
$K\backslash T$	30	60	90	120	150	180	210	240	270	300	330	360
$S_0(1-0.2)$	1,984.643	2,372.639	2,679.990	2,939.135	3,165.324	3,367.079	3,549.728	3,716.873	3,871.086	4,014.285	4,147.945	4,273.234
$S_0(1-0.16)$	1,795.751	2,211.874	2,535.195	2,805.692	3,040.802	3,249.963	3,438.978	3,611.722	3,770.944	3,918.678	4,056.485	4,185.595
$S_0(1-0.12)$	1,620.671	2,061.193	2,398.584	2,679.240	2,922.427	3,138.357	3,333.231	3,511.161	3,675.043	3,827.015	3,968.710	4,101.413
$S_0(1-0.08)$	1,459.268	1,920.193	2,269.755	2,559.400	2,809.850	3,031.938	3,232.190	3,414.912	3,583.124	3,739.052	3,884.391	4,020.471
$S_0(1-0.04)$	1,311.222	1,788.440	2,148.305	2,445.810	2,702.742	2,930.405	3,135.577	3,322.717	3,494.949	3,654.566	3,803.317	3,942.572
S_0	1,176.059	1,665.482	2,033.841	2,338.125	2,600.795	2,833.478	3,043.136	3,234.341	3,410.296	3,573.351	3,725.297	3,867.535
$S_0(1+0.04)$	1,053.180	1,550.854	1,925.980	2,236.013	2,503.716	2,740.895	2,954.627	3,149.563	3,328.963	3,495.218	3,650.153	3,795.194
$S_0(1+0.08)$	941.897	1,444.090	1,824.353	2,139.163	2,411.232	2,652.411	2,869.829	3,068.181	3,250.762	3,419.994	3,577.722	3,725.396
$S_0(1+0.12)$	841.460	1,344.729	1,728.602	2,047.277	2,323.085	2,567.797	2,788.536	2,990.007	3,175.521	3,347.517	3,507.857	3,658.001
$S_0(1+0.16)$	751.087	1,252.319	1,638.388	1,960.074	2,239.033	2,486.841	2,710.555	2,914.865	3,103.078	3,277.639	3,440.416	3,592.880
$S_0(1+0.2)$	669.982	1,166.420	1,553.385	1,877.290	2,158.850	2,409.341	2,635.708	2,842.592	3,033.284	3,210.222	3,375.274	3,529.913

Table A.14. European Put Bitcoin Prices Under the Heston-Nandi GARCH Model using the Variance-Dependent Pricing Kernel with $\xi = 300$

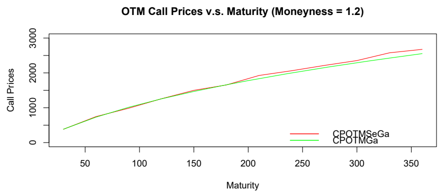
$K\backslash T$	30	60	90	120	150	180	210	240	270	300	330	360
$S_0(1-0.2)$	477.457	856.039	1,153.991	1,403.752	1,620.572	1,812.972	1,986.282	2,144.102	2,289.005	2,422.907	2,547.286	2,663.309
$S_0(1-0.16)$	587.645	993.883	1,307.335	1,567.980	1,793.252	1,992.591	2,171.799	2,334.752	2,484.197	2,622.171	2,750.233	2,869.613
$S_0(1-0.12)$	711.645	1,141.812	1,468.865	1,739.198	1,972.078	2,177.718	2,362.319	2,529.992	2,683.632	2,825.379	2,956.865	3,079.374
$S_0(1-0.08)$	849.322	1,299.421	1,638.174	1,917.028	2,156.703	2,368.033	2,557.545	2,729.543	2,887.049	3,032.286	3,166.952	3,292.376
$S_0(1-0.04)$	1,000.357	1,466.278	1,814.864	2,101.109	2,346.798	2,563.235	2,757.200	2,933.150	3,094.208	3,242.671	3,380.285	3,508.420
S_0	1,164.273	1,641.929	1,998.540	2,291.094	2,542.053	2,763.042	2,961.025	3,140.574	3,304.891	3,456.326	3,596.671	3,727.326
$S_0(1+0.04)$	1,340.475	1,825.911	2,188.819	2,486.653	2,742.176	2,967.194	3,168.784	3,351.597	3,518.893	3,673.064	3,815.933	3,948.928
$S_0(1+0.08)$	1,528.272	2,017.756	2,385.331	2,687.473	2,946.893	3,175.444	3,380.253	3,566.016	3,736.028	3,892.711	4,037.910	4,173.073
$S_0(1+0.12)$	1,726.915	2,217.005	2,587.720	2,893.257	3,155.948	3,387.564	3,595.227	3,783.643	3,956.122	4,115.104	4,262.450	4,399.622
$S_0(1+0.16)$	1,935.622	2,423.205	2,795.645	3,103.725	3,369.098	3,603.342	3,813.514	4,004.302	4,179.015	4,340.097	4,489.417	4,628.444
$S_0(1+0.2)$	2,153.598	2,635.915	3,008.781	3,318.611	3,586.117	3,822.577	4,034.934	4,227.830	4,404.556	4,567.550	4,718.681	4,859.420



(a) Panel A

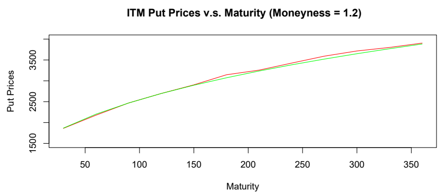


(b) Panel B

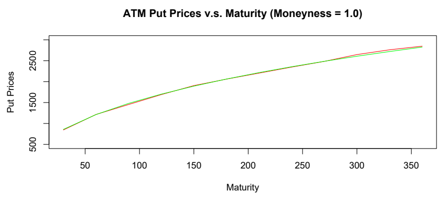


(c) Panel C

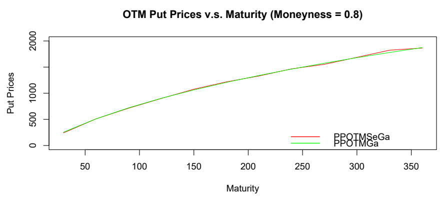
Figure B.1. European Bitcoin Call Prices against Maturity Under the SETAR-Heston-Nandi-GARCH Model and the Heston-Nandi GARCH Model:
Panels A, B and C plot the European Bitcoin Call Prices against Maturity (T) for ITM Options (Moneyness $K/S_0 = 0.8$), ATM Options (Moneyness $K/S_0 = 1.0$) and OTM Options (Moneyness $K/S_0 = 1.2$), respectively.



(a) Panel A

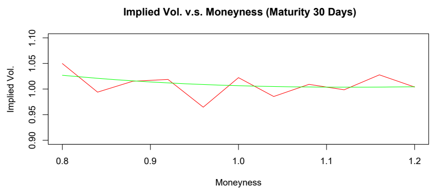


(b) Panel B

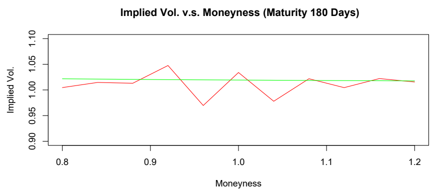


(c) Panel C

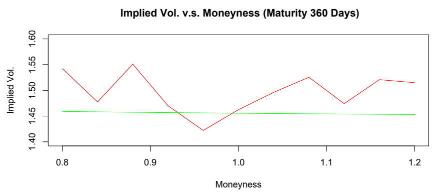
Figure B.2. European Bitcoin Put Prices against Maturity Under the SETAR-Heston-Nandi-GARCH Model and the Heston-Nandi GARCH Model: Panels A, B and C plot the European Bitcoin Put Prices against Maturity (T) for ITM Options (Moneyness $K/S_0 = 1.2$), ATM Options (Moneyness $K/S_0 = 1.0$) and OTM Options (Moneyness $K/S_0 = 0.8$), respectively.



(a) Panel A

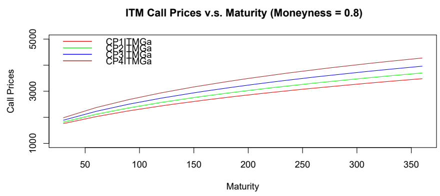


(b) Panel B

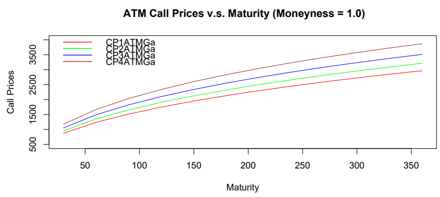


(c) Panel C

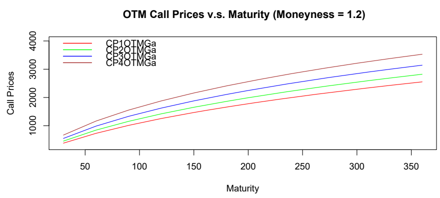
Figure B.3. Black-Scholes Implied Volatilities for European Bitcoin Call Options Under the SETAR-Heston-Nandi-GARCH Model and the Heston-Nandi GARCH Model. Panels A, B and C plot the Black-Scholes implied volatilities of European call prices from the SETAR-Heston-Nandi-GARCH and Heston-Nandi GARCH models against moneyness (K/S_0) for different maturities, say 30 days (short maturity), 180 days (medium maturity) and 360 days (long maturity), respectively.



(a) Panel A

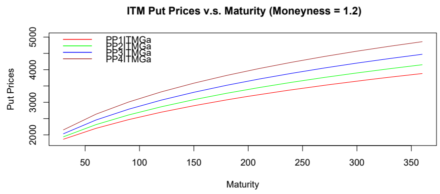


(b) Panel B

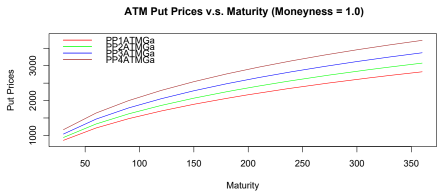


(c) Panel C

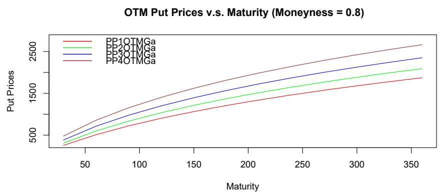
Figure B.4. European Bitcoin Call Prices against Maturity Under the Heston-Nandi GARCH Model Using the Variance-Dependent Pricing Kernel: Panels A, B and C plot the European Bitcoin Call Prices against Maturity (T) for ITM Options (Moneyness $K/S_0 = 0.8$), ATM Options (Moneyness $K/S_0 = 1.0$) and OTM Options (Moneyness $K/S_0 = 1.2$), respectively, where $\xi = 0, 100, 200, 300$.



(a) Panel A

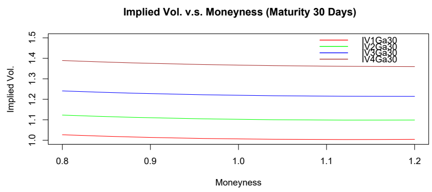


(b) Panel B

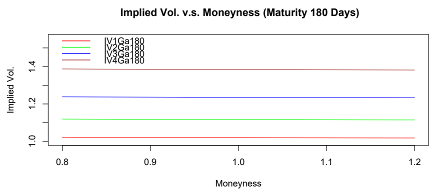


(c) Panel C

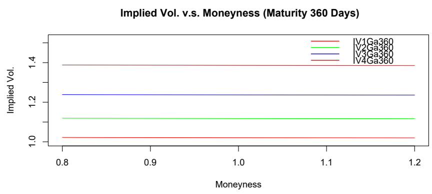
Figure B.5. European Bitcoin Put Prices against Maturity Under the Heston-Nandi GARCH Model Using the Variance-Dependent Pricing Kernel: Panels A, B and C plot the European Bitcoin Put Prices against Maturity (T) for ITM Options (Moneyness $K/S_0 = 1.2$), ATM Options (Moneyness $K/S_0 = 1.0$) and OTM Options (Moneyness $K/S_0 = 0.8$), respectively, where $\xi = 0, 100, 200, 300$.



(a) Panel A



(b) Panel B



(c) Panel C

Figure B.6. Black-Scholes Implied Volatilities for European Bitcoin Call Options under the Heston-Nandi GARCH Model using the Variance-Dependent Pricing Kernel. A, B and C plot the Black-Scholes implied volatilities of European call prices from the GARCH model using the variance-dependent pricing kernel with $\xi = 0, 100, 200, 300$ against moneyness (K/S_0) for different maturities, say 30 days (short maturity), 180 days (medium maturity) and 360 days (long maturity), respectively.